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SIMPLE GROUP ACTIONS ON HYPERBOLIC RIEMANN SURFACES OF LEAST AREA

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It is shown that if a simple group G acts conformally on a hyperbolic surface of least area (or alternatively, on a Riemann surface of least genus $\sigma \geq 2$), then G is normal in $\mathrm{Aut}(S)$ and the map $\mathrm{Aut}(S) \to \mathrm{Aut}(G)$ induced by conjugation is injective. For the preponderance of these minimal actions the group $\mathrm{Aut}(S)/G$ is isomorphic to a subgroup of Σ_3. It is shown how to compute $\mathrm{Aut}(S)$ purely in terms of the group-theoretic structure of G, in these cases. As examples and as part of the proof, the minimal actions and the groups $\mathrm{Aut}(S)$ are completely worked out for A_5, $\mathrm{SL}_3(3)$, M_{11} and M_{12}.

1. Introduction. If G is a finite group, then G can act as a group of conformal (i.e. biholomorphic) automorphisms of a closed Riemann surface for infinitely many genera. Several authors, **[C]**, **[G-S1]**, **[G-S2]**, **[H1]**, **[M]**, **[T]** and **[W]**, have considered the question of determining the least genus of a surface on which a given group can act conformally. Tucker **[T]** calls this least genus the *strong symmetric genus* of the group, though we will adopt the terminology of H. Glover and call this least genus the *action genus*. Actions on such surfaces we shall call *genus actions*. Conder **[C]** has determined the action genera of all the alternating groups. Glover and Sjerve **[G-S1]**, **[G-S2]** have determined the action genera for $\mathrm{PSL}_2(p^k)$, p a prime. Harvey **[H1]** and McLachlan **[M]** have worked out procedures to easily determine genus actions of cyclic and abelian groups, respectively.

If the surface S has genus $\sigma \geq 2$, then $\mathrm{Aut}(S)$ is finite and, according to Hurwitz's famous theorem, $|\mathrm{Aut}(S)| \leq 84(\sigma - 1)$. In this paper we consider the following question for simple groups (all our simple groups are non-abelian) acting on surfaces with genus ≥ 2.

> If the simple group G acts conformally on the closed Riemann surface S, of least genus, then how large a subgroup of $\mathrm{Aut}(S)$ is G?

It turns out that G is a normal subgroup of $\mathrm{Aut}(S)$ of very small index. This is our main result, Theorem 1.1 below. On the other hand for $e \geq 2$, and a prime $p > 2e + 1$ there are genus actions of $G = (\mathbf{Z}_p)^{2e}$ on surfaces S of genus $\sigma = (e-1)p^{2e} + 1$ such that