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A SPECTRAL THEORY FOR SOLVABLE LIE ALGEBRAS OF OPERATORS

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The main objective of this paper is to develop a notion of joint spectrum for complex solvable Lie algebras of operators acting on a Banach space, which generalizes Taylor joint spectrum (T.J.S.) for several commuting operators.

I. Introduction. We briefly recall the definition of Taylor spectrum. Let $\bigwedge(\mathbb{C}^n)$ be the complex exterior algebra on n generators $e_1, \ldots, e_n$, with multiplication denoted by $\wedge$. Let $\dot{E}$ be a Banach space and $a = (a_1, \ldots, a_n)$ be a mutually commuting n-tuple of bounded linear operators on E(m.c.o.). Define $\bigwedge_k^n(E) = \bigwedge_k(\mathbb{C}^n) \otimes_{\mathbb{C}} E$, and for $k \geq 1$, D_{k-1} by:

$$D_{k-1}\colon \bigwedge_k^n(E) \to \bigwedge_{h-1}^n(E)$$

$$\begin{aligned} &D_{k-1}(x \otimes e_{i_1} \wedge \cdots \wedge e_{i_k}) \\ &\quad = \sum_{j=1}^{k} (-1)^{j+1} x \cdot a_{i_j} \otimes \cdots \otimes e_{i_1} \wedge \cdots \wedge \tilde{e}_{i_j} \wedge \cdots \wedge e_{i_k} \end{aligned}$$

where $\sim$ means deletion. Also define $D_k = 0$ for $k \leq 0$.

It is easily seen that $D_k D_{k+1} = 0$ for all k, that is, $\{\bigwedge_k^n(E), D_k\}_{k \in Z}$ is a chain complex, called the Koszul complex associated with a and E and denoted by $R(E, a)$. The n-tuple a is said to be invertible or nonsingular on E, if $R(E, a)$ is exact, i.e., $\operatorname{Ker} D_k = \operatorname{ran} E_{k+1}$ for all k. The Taylor spectrum of a on E is $\operatorname{Sp}(a, E) = \{\lambda \in \mathbb{C}^n\colon a - \lambda$ is not invertible$\}$.

Unfortunately, this definition depends very strongly on $a_1, \ldots, a_n$ and not on the vector subspace of $L(E)$ generated by then $(= \langle a \rangle)$.

As we consider Lie algebras, and then naturally involve geometry, we are interested in a geometrical approach to spectrum which depends on L rather than on a particular set of operators.

This is done in II. Given a solvable Lie subalgebra of $L(E)$, L, we associate to it a set in L^*, $Sp(L, E)$.