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DETERMINANT IDENTITIES

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A number of determinants are evaluated in closed form including

$$\det\left(\binom{i+j+x}{2i-j}+\binom{i+j+y}{2i-j}\right)_{0\le i,j\le n-1}.$$

1. Introduction. In one of their series of papers on plane partitions and related questions, Mills, Robbins and Rumsey [**9**; p. 53] prove the following determinant formula.

$$m_n(x)=\det\left(\binom{i+j+x}{2i-j}\right)_{0\le i,j\le n-1}=\frac{1}{2^n}\prod_{k=0}^{n-1}\Delta_{2k}(2x), \tag{1.1}$$

where $\Delta_0(u)=2$ and for $j>0$

$$\Delta_{2j}(u)=\frac{(u+2j+2)_j(\frac{1}{2}u+2j+\frac{3}{2})_{j-1}}{(j)_j(\frac{1}{2}u+j+\frac{3}{2})_{j-1}} \tag{1.2}$$

with

$$(A)_j=A\cdot(A+1)\cdots(A+j-1). \tag{1.3}$$

Our object here is primarily to prove the following generalization of (1.1).

THEOREM 1. *Let*

$$M_n(x,y)=\det\left(\binom{i+j+x}{2i-j}+\binom{i+j+y}{2i-j}\right)_{0\le i,j\le n-1}; \tag{1.4}$$

$$N_n(x,y)=\det\left(\frac{2}{x+1-y}\left\{\binom{i+j+x+1}{2i-j+1}-\binom{i+j+y}{2i-j+1}\right\}\right)_{0\le i,j\le n-1}. \tag{1.5}$$

Then

$$M_n(x,y)=N_n(x,y)=\prod_{k=0}^{n-1}\Delta_{2k}(x+y). \tag{1.6}$$