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ANY KNOT COMPLEMENT COVERS AT MOST ONE KNOT COMPLEMENT

SHICHENG WANG AND YING-QING WU

It follows from Culler, Gordon, Luecke and Shalen's Cyclic Surgery Theorem that any knot complement is covered by at most two knot complements. Gonzales-Acuna and Whitten proved a result on the other direction: A given knot complement can cover at most finitely many knot complements. This paper is to show that the best possible result in this direction holds: A given knot complement can nontrivially cover at most one knot complement. Moreover, if the knot is not a torus knot, then the covering map is unique up to equivalence.

Given a 3-manifold M, there are generically infinitely many manifolds which cover M. However, if we are restricted to the category of knot complements, the situation is quite different. It can be shown (see Lemma 1 and below) that if the complement $E(K)$ of a knot K is n-fold covered by some knot complement, then the covering is cyclic, and K admits a cyclic surgery, i.e. a Dehn surgery such that the fundamental group of the resulting manifold is a cyclic group $\mathbf{Z}_n$. It follows from the Cyclic Surgery Theorem of **[CGLS]** that if K is not a torus knot, then there are at most two such coverings. The situation is also clear if K is a torus knot: By a theorem of Moser **[M]**, a Dehn surgery on a (p, q) torus knot $T(p, q)$ is a cyclic surgery if and only if the surgery coefficient is $(kpq \pm 1)/k$ for some k. Now the $kpq \pm 1$ fold cyclic covering of the complement $E(K)$ of K is always homeomorphic to $E(K)$ itself, with possibly an orientation reversing homeomorphism. So $E(K)$ is only covered by one knot complement, although there are infinitely many different covering maps.

In this paper we will study a closely related problem: How many knot complements are nontrivially covered by a given knot complement $E(K)$? The problem was studied by Gonzales-Acuna and Whitten in **[GW]**, where they proved that a knot complement covers at most finitely many knot complements up to homeomorphism. The main result of this paper is

THEOREM 1. *The complement $E(K)$ of any knot K can nontrivially cover at most one knot complement $E(K')$ up to homeomorphism.*