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THE DUAL PAIR $(U(1), U(1))$ OVER A p-ADIC FIELD

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We find an explicit decomposition for the metaplectic representation restricted to either member of the dual reductive pair $(U(1), U(1))$ in $\widetilde{\mathrm{SL}}(2, F)$, where F is a p-adic field, with p odd.

1. Introduction and preliminaries. Let F be a p-adic field of odd residual characteristic with q being the order of the residue class field. Let $\mathscr{O}$ be the ring of integers, $\mathscr{P}$ the prime ideal, $\mathscr{U}$ the units, π a prime element, and ν the valuation on F. Let $G = \mathrm{SL}(2, F)$.

For $\sigma = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) \in G$, let $x(\sigma) = c$ if $c \neq 0$, and let $x(\sigma) = d$ if $c = 0$. Define a 2-cocycle on G by

$$\alpha(g_1, g_2) = (x(g_1), x(g_2))(-x(g_1)x(g_2), x(g_1 g_2)).$$

This cocycle determines a nontrivial 2-sheeted covering group $\widetilde{G}$ of G **[G1]**.

Let E be a quadratic extension of F, and $x \mapsto \overline{x}$ the Galois action. The group $U(1)$ which preserves the Hermitian form $(x, y) \mapsto x\overline{y}$ on E is isomorphic to the group N^1 of norm one elements in E. The pair of subgroups $(U(1), U(1))$ of SL(2) form a dual reductive pair **[H]**. This dual pair is one of the simplest examples over a p-adic field. Some other basic examples of dual reductive pairs are discussed in **[G2]**. In this paper we determine the decomposition of the oscillator representation of $\widetilde{G}$ upon restriction to $U(1) \subset \widetilde{G}$.

The results in this paper have recently been applied by Rogawski to the problem of calculating the multiplicities of certain automorphic representations π of $U(\mathbf{A})$ in the discrete spectrum of $L^2(U(k)\backslash U(\mathbf{A}))$, where U is a unitary group in 3 variables defined relative to a quadratic extension of number fields K/k **[R1, R2]**. I would like to thank Rogawski for several stimulating conversations and for encouraging me to publish this paper.

Let τ be a character of F. Choose a normalized measure μ so that $\mu(\mathscr{O}) = q^{\frac{\omega(\tau)}{2}}$, where $\omega(\tau)$ is the conductor of τ. Denote this measure by $d_\tau x$. Then if we define the Fourier transform on $S(F)$, the space of locally compact functions on F with compact support,