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STATIONARY SURFACES IN MINKOWSKI SPACES, I. A REPRESENTATION FORMULA

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Gu has derived a representation formula for stationary surfaces in the 3-dimensional Minkowski space using the Legendre transformation. By a different method, we generalize his results to any Minkowski spaces.

1. Introduction. Given a differentiable map $f\colon (M_1, g_1) \to (M_2, g_2)$ between two manifolds with nondegenerate (definite or indefinite) metrics, its energy is defined as

$$E(f) = \int_{M_1} |df|^2 \, d\,\mathrm{vol}_{g_1}.$$

A critical point of this energy functional is called a *harmonic map.* Let (M_2, g_2) be any Minkowski space, (M_1, g_1) an immersed submanifold of M_2 such that the induced metric g_1 is nondegenerate except possibly on a subset of codimension at least one. Let $f\colon M_1 \to M_2$ be the inclusion map. Then its energy is still well defined. If f is a critical point for the energy (i.e., the mean curvature vanishes), M_1 is called a *stationary submanifold* in the Minkowski space M_2; if M_1 has real dimension two, then we call it a *stationary surface,* since in general it is not extremal—neither minimal nor maximal. Although minimal surface theory in Euclidean spaces has been studied for hundreds of years, the analogous stationary surface theory in Minkowski spaces only has a very short history. Suppose p is a point on the surface S in a Minkowski space. We call p an *elliptic, hyperbolic, parabolic* point if the induced metric on S at p is definite, indefinite but nondegenerate, degenerate respectively. A powerful tool to study the stationary surface is to use isothermal coordinates, whose existence is well-known. At a hyperbolic point, if the metric is written as $ds^2 = f(x, y)\,dx\,dy$, then (x, y) are called *characteristic coordinates.* This paper is inspired by Gu's work **[1]** and **[2]**, of which many ideas are carried on here.

2. Representation formula. We denote by $\mathbb{R}^{m,n}$ the Minkowski space of signature (m, n), i.e., the vector space R^{m+n} with the metric