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SHANYU JI

A compact complex manifold M is Moishezon if and only if there exists an integral closed positive $(1, 1)$-current ω such that $\omega \geq \varepsilon\sigma$ and ω is smooth outside an analytic subvariety.

1. Introduction. Given a Moishezon manifold M, it is well known (cf. [**Mo**], [**W**]) that there is a bimeromorphic morphism $\pi: \widetilde{M} \to M$ such that the manifold $\widetilde{M}$ is projective algebraic. Let $\tilde{\omega}$ be Kähler form on $\widetilde{M}$ with $[\tilde{\omega}] \in H^2(\widetilde{M}, \mathbf{Z})$. Then the pushforward current $\omega = \pi_* \tilde{\omega}$ is a d-closed current on M such that

(i) $[\omega] \in H^2(M, \mathbf{Z})$;

(ii) ω is smooth on $M - S$, where S is some proper analytic subset in M;

(iii) $\omega \geq \varepsilon\sigma$ in the sense of currents, where $\varepsilon > 0$ is some real number and σ is a fixed positive definite $(1, 1)$-form (not necessarily d-closed) on M.

Conversely, we shall prove the following

THEOREM 1.1. *Let M be a compact complex manifold of dimension n. Then M is Moishezon if and only if there exists a d-closed* $(1, 1)$*-current ω on M such that the conditions* (i), (ii) *and* (iii) *above are satisfied.*

In fact, the above theorem is a weak version of a general conjecture of Shiffman [**J**] which asked: whether a compact complex manifold N is Moishezon if and only if there exists a d-closed $(1, 1)$-current satisfying the conditions (i) and (iii) above. The conjecture is to generalize the well-known Kodaira embedding theorem in terms of currents and it is still unknown. Some partial results have been obtained [**J**]: if M is complex torus, Shiffman's conjecture is true; if S is a set of isolated points, Theorem 1.1 follows from an extension theorem of Miyaoka [**M**]; if S is special in some sense, Theorem 1.1 is also true. All of these results are proved by smoothing of currents technique, and depends on a fact that the top degree Chern number $(c_1([\omega])^n, M) > 0$. However, it is easy to find an example of a current ω satisfying (i), (ii)