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ON ISOTROPIC SUBMANIFOLDS AND EVOLUTION OF QUASICAUSTICS

STANISLAW JANECZKO [1]

We study classification problems for generic isotropic submanifolds. The classification list of simple and unimodal singularities is obtained and the generic evolutions of quasicaustics in small dimension are classified. Examples encountered in geometric optics are presented.

0. Introduction and preliminaries. Let X be a manifold, and ω be a 2-form on X. The pair (X, ω) is called a symplectic manifold if ω is closed, i.e. $d\omega = 0$ and nondegenerate [AM]. The representative model of a symplectic manifold is a cotangent bundle T^*M, endowed with the canonical 2-form $\omega_M = d\vartheta_M$, where the 1-form ϑ_M on T^*M (Liouville form) is defined by

$$\langle u, \vartheta_M \rangle = \langle T\pi_M(u), \tau_{T^*M}(u) \rangle, \quad \text{for each } u \in TT^*M.$$

The mapping $T\pi_M$ is the tangent mapping of $\pi_M: T^*M \to M$ and $\tau_{T^*M}: TT^*M \to T^*M$ is the tangent bundle projection. If (q_i) are local coordinates introduced in M, and (p_i, q_i) are corresponding local coordinates in T^*M then ω_M has the normal (Darboux) form $\omega_M = \sum_{i=1}^n dp_i \wedge dq_i$ [We].

We recall that a submanifold $C \subset (X, \omega)$ is *coisotropic* if, at each $x \in C$, the symplectic polar of T_xC defined by

$$C_x^\perp = \{v \in T_xX : \langle v \wedge u, \omega \rangle = 0 \text{ for every } u \in T_xC\}$$

is contained in T_xC. By $\langle v \wedge u, \omega \rangle$ we denote the evaluation of ω on the pair of vectors $v, u \in T_xX$. If $C_x^\perp = T_xC$ for each $x \in C$ then C is called the *Lagrangian submanifold* of X. In this case $\omega|_C = 0$, and $\dim C = \frac{1}{2}\dim X$. We see that $\dim C_x^\perp = \operatorname{codim} C$ and $\{C_x^\perp\}$ forms the characteristic distribution of $\omega|_C$. Thus the distribution $D = \bigcup_{x \in C} C_x^\perp$ is involutive. Maximal connected integral manifolds of D are called *bicharacteristics*. They form the characteristic foliation of C (cf. [AM]). D represents the generalized Hamiltonian system

[1] On leave from Mathematics Institute, Technical University of Warsaw, Pl. Politechniki 1, 00-661 Warsaw, Poland.