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SOME APPLICATIONS OF BELL'S THEOREM TO WEAKLY PSEUDOCONVEX DOMAINS

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In this paper, we study some problems of holomorphic maps in weakly pseudoconvex domains. For example, we consider the boundary version of the rigidity properties both for automorphisms and for self-holomorphic maps, the existence of the interior fixed points for some automorphisms, and the minimal property of the rank of the Levi form at boundary orbit accumulation points of the automorphism groups.

0. Introduction. Suppose Ω is a bounded domain in $\mathbf{C}^n$, and σ_n $(n = 1, 2, \ldots)$, $\sigma \in \mathrm{Aut}(\Omega)$. It is a classical theorem of Cartan which states that if σ_n converges to σ pointwise, then $\sigma_n \to \sigma$ in the topology of $C^\infty(K)$ for any $K \Subset \Omega$.

A natural question to ask is under what circumstances one can conclude furthermore that $\sigma_n \to \sigma$ in $C^\infty(\overline{\Omega})$.

Greene-Krantz [4] solved this problem affirmatively for Ω strongly pseudoconvex by using Fefferman's work on the asymptotic expansion of Bergman kernel function. More recently, Bell [2] proved the following

THEOREM (*Bell*). *Let Ω be a bounded pseudoconvex domain of finite type (in the sense of D'angelo), and let σ_n, $\sigma \in \mathrm{Aut}(\Omega)$ be such that $\sigma_n \to \sigma$ on compacta. Then $\sigma_n \to \sigma$ in $C^\infty(\overline{\Omega})$. Moreover, if $\sigma_n \to p \in \partial\Omega$ (i.e., $\{\sigma_n\}$ converges to the constant map $c(z) \equiv p$ on compacta), and if $\sigma_n^{-1} \to q \in \partial\Omega$, then $\sigma_n \to p$ in $C^\infty(\overline{\Omega} - \{q\})$.*

As noted in Bell's paper, the first part of this theorem is actually valid for pseudoconvex domains which satisfy condition R. It would be very interesting to extend it to all smooth pseudoconvex domains. However, in this paper, we are only concerned with some applications of this important result and their related developments. For example, the following kinds of problems will be studied:

(A) *The boundary version of the rigidity theorem.* A fundamental theorem of Cartan says that if Ω is a bounded domain in $\mathbf{C}^n$, $p \in \Omega$,