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L^n SOLUTIONS OF THE STATIONARY AND NONSTATIONARY NAVIER-STOKES EQUATIONS IN R^n

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It is shown that the Navier-Stokes equations in the whole space R^n $(n \geq 3)$ admit a unique small stationary solution which may be formed as a limit of a nonstationary solution as $t \to \infty$ in L^n-norms.

0. Introduction. As is well known, the existence of solutions to the exterior stationary Navier-Stokes equations was studied by Finn [2, 3], and small solutions from Finn [2, 3] may be formed as limits of nonstationary solutions as time $t \to \infty$ in local or global L^2-norms (cf. Heywood [9, 10], Galdi and Rionero [6], Miyakawa and Sohr [16], Borchers and Miyakawa [1]) and in the norms of other function spaces (cf. Heywood [11], Musuda [14]). However, it is still unknown even in the case of whole spaces whether or not

$$\|v(t)-w\|_n + t^{1/2}\|Dv(t)-Dw\|_n + t^{1/2}\|v(t)-w\|_\infty \to 0 \quad \text{as } t \to \infty, \tag{0.1}$$

provided that w and v are, respectively, the solutions to the stationary Navier-Stokes equations

$$-\Delta w + (w\cdot D)w + d\overline{p} = f, \quad D\cdot w = 0 \quad \text{in } R^n \tag{0.2}$$

and the nonstationary Navier-Stokes equations

$$\begin{gathered} v_t - \Delta v + (v\cdot D)v + D\overline{\overline{p}} = f, \quad D\cdot v = 0 \quad \text{in } R^n \times (0,\infty), \\ v(0) = v_0 \quad \text{in } R^n. \end{gathered} \tag{0.3}$$

Here and in what follows, $n \geq 3$ denotes the space dimension, $\overline{p}$ and $\overline{\overline{p}}$ represent the pressures associated with w and v, respectively, $D =$ the gradient, $f = f(x)$ is a prescribed function, the dot $\cdot$ denotes the scalar product in R^n, and $\|\cdot\|_r$ denotes the norm of the Lebesgue space $L^r = L^r(R^n; R^n)$.

The purpose of the paper is to show that (0.2) and (0.3) admit small regular solutions w and $v(t)$ in L^n, respectively, such that (0.1) is valid. The problem above is, as usual, said to be a stability problem