PRIMITIVE SUBALGEBRAS
OF COMPLEX LIE ALGEBRAS. I.
PRIMITIVE SUBALGEBRAS
OF THE CLASSICAL COMPLEX LIE ALGEBRAS

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This paper lists all the primitive reductive non-maximal subalgebras of simple complex classical Lie algebras.

The classification problem of primitive actions of connected Lie groups on manifolds, which was raised yet by Sophus Lie [11], and the problem of finding the maximal subgroups of connected Lie groups boil down to the classification problem of primitive subalgebras of Lie algebras (Lie groups and algebras are assumed to be finite-dimensional).

The classification of maximal connected subgroups of connected Lie groups reduces to the classification of maximal subalgebras of Lie algebras, which was obtained by the early fifties by V. V. Morozov [13], [14], F. I. Karpelevich [9], A. Borel and J. de Siebenthal [7], and E. B. Dynkin [4], [5].

Any maximal subalgebra of an algebra $\mathfrak{G}$, which contains no proper ideal of $\mathfrak{G}$, is primitive. In 1972, M. Golubitsky [7] found primitive non-maximal subalgebras among maximal-rank subalgebras of simple classical complex Lie algebras. Later M. Golubitsky and B. Rothschild [8] found primitive non-maximal subalgebras among maximal-rank subalgebras of simple exceptional complex Lie algebras.

M. Golubitsky [7] proved that primitive subalgebras of non-reductive primitive subalgebras of simple complex algebras are maximal. Thus the problem of finding primitive subalgebras of complex Lie algebras was reduced to the case of reductive subalgebras of simple Lie algebras.

This work provides the final solution of the question of primitive subalgebras of the classical complex Lie algebras; in fact, we list all the primitive reductive non-maximal subalgebras of simple complex classical Lie algebras.

This work's main results have been published in [3]. Note that three cases of primitive non-maximal subalgebras were left out in [3].