PACIFIC JOURNAL OF MATHEMATICS
Vol. 158, No. 2, 1993

FIXED POINTS OF BOUNDARY-PRESERVING MAPS OF SURFACES

ROBERT F. BROWN AND BRIAN J. SANDERSON

Let X be a compact 2-manifold with nonempty boundary ∂X. Given a boundary-preserving map $f: (X, \partial X) \to (X, \partial X)$, let $MF_\partial[f]$ denote the minimum number of fixed points of all boundary-preserving maps homotopic to f as maps of pairs and let $N_\partial(f)$ be the relative Nielsen number of f in the sense of Schirmer [S]. Call X *boundary-Wecken*, bW, if $MF_\partial[f] = N_\partial(f)$ for all boundary-preserving maps of X, *almost bW* if $MF_\partial[f] - N_\partial(f)$ is bounded for all such f, and *totally non-bW* otherwise. We show that if the euler characteristic of X is non-negative, then X is bW. On the other hand, except for a relatively small number of cases, we demonstrate that the 2-manifolds of negative euler characteristic are totally non-bW. For one of the remaining cases, the *pants surface* P, we use techniques of transversality theory to examine the fixed point behavior of boundary-preserving maps of P, and show that P is almost bW.

1. Introduction. Throughout this paper, we will be working in the setting of compact manifolds. Given a map $f: X \to X$ of a compact manifold X, we denote the Nielsen number of f by $N(f)$ and let $MF[f]$ be the minimum number of fixed points of all maps homotopic to f. The manifold X is said to be *Wecken* if $MF[f] = N(f)$ for all maps $f: X \to X$. Wecken [W] proved that all n-manifolds are Wecken for $n \geq 3$ and Jiang [J] proved that a 2-manifold is Wecken if and only if its euler characteristic is non-negative. The interval is obviously Wecken and it is a classical result that the circle is Wecken.

Now suppose that the manifold X has nonempty boundary ∂X and that f is boundary-preserving, that is, f maps ∂X to itself so f is a map of pairs $f: (X, \partial X) \to (X, \partial X)$. We denote the relative Nielsen number by $N_\partial(f)$ and write $MF_\partial[f]$ for the minimum number of fixed points of all maps homotopic to f as maps of pairs. We say that a manifold X with nonempty boundary is *boundary-Wecken*, abbreviated bW, if $MF_\partial[f] = N_\partial(f)$ for all maps $f: (X, \partial X) \to (X, \partial X)$. It is obvious that the interval is bW and Schirmer [S] proved that all n-manifolds are bW for $n \geq 4$. The purpose of this paper is to investigate the bW property for boundary-preserving maps of 2-manifolds. We begin, however, with a remark