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ON THE EXTENSION OF LIPSCHITZ FUNCTIONS FROM BOUNDARIES OF SUBVARIETIES TO STRONGLY PSEUDOCONVEX DOMAINS

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In this paper, we study the principal value integral on boundaries of subvarieties in strongly pseudoconvex domains and using it, we give a condition for the extendability of Lipschitz functions.

Introduction. Let D be a strongly pseudoconvex domain in C^n with C^∞ boundary. Henkin [6] and Ramírez [12] obtained independently the support function $g(\zeta, z)$ for D which depends holomorphically on z, and then, using this support function, they obtained the integral formula for holomorphic functions in $\overline{D}$. On the other hand, Stout [14], when $p = 1$, and then, Hatziafratis [5], when p is arbitrary, obtained the integral formula for a certain subvariety V of codimension p in D. By using the support function $g(\zeta, z)$ and the integral formula for V, we can obtain the kernel $\Omega(\zeta, z)$ for $(\zeta, z) \in \partial V \times \overline{D}$. In this paper, we shall define the principal value integral P.V. $\int_{\partial V} f(\zeta)\Omega(\zeta, z)$ for a Lipschitz function f on ∂V and $z \in \partial V$. The definition of the principal value integral is the same as that of Alt [2] when $V = D$ (cf. Dolbeault [4]). By using the principal value integral we can give the condition for a Lipschitz function on ∂V to be the boundary value of a function that is holomorphic in D and continuous on $\overline{D}$. Finally we end the introduction by giving an example which shows that the Lipschitz continuity is necessary in order to define the principal value integral.

EXAMPLE. Define $\varphi \in C^\infty(0, \infty)$ such that

$$\varphi(\theta) = \begin{cases} 1 & \text{if } 0 < \theta \le \frac{\pi}{4}, \\ 0 & \text{if } \theta \ge \frac{\pi}{2}. \end{cases}$$

Extend φ to an odd function on $R|\{0\}$. Let D be the unit disc in $\mathbb{C}$ and f be a function on ∂D such that

$$f(e^{i\theta}) = \begin{cases} \frac{\varphi(\theta)}{\log|\theta|} & \text{if } 0 < |\theta| \le \pi, \\ 0 & \text{if } \theta = 0. \end{cases}$$