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THE INTRINSIC GROUP OF MAJID'S BICROSSPRODUCT KAC ALGEBRA

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A precise description of the intrinsic group of a Kac algebra considered in the recent work of Majid associated with a modular matched pair is given. By using the result, a detailed computation is done to produce an interesting pair of nonisomorphic Kac algebras.

0. Introduction. In [M1], [M2] and [M3], Majid studied the notion of a matched pair of locally compact groups and their actions. He exhibited plenty of examples of such pairs, relating them to solutions to the classical Yang-Baxter equations. Among other things, he showed in [M3] that every matched pair gives rise to two involutive Hopf-von Neumann algebras which are not commutative or cocommutative except in the trivial case. Moreover, he proved that, if a matched pair is modular in his sense, then the resulting von Neumann algebras turn out to be Kac algebras, dual to each other. (See [E&S] and §1 for definitons of an involutive Hopf-van Neumann algebra and a Kac algebra.) He called these algebras bicrossproduct Kac algebras. Thus his result furnishes abundant examples of nontrivial Kac algebras. It should be noted that, through his construction, one can even obtain a noncommutative, noncocommutative, self-dual Kac algebra. All of these would suggest that matched pairs of groups and bicrossproduct algebras deserve a further detailed investigation.

In the meantime, the notion of the intrinsic group $G(\mathbf{K})$ of a Kac algebra $\mathbf{K}$ was introduced by Schwartz in [S]. Roughly speaking, it consists of "group-like" elements of the given Kac algebra. Thus the group $G(\mathbf{K})$ can be considered as a natural kind of invariant attached to each Kac algebra $\mathbf{K}$. In fact, if a Kac algebra $\mathbf{K}$ is either commutative or cocommutative, then the intrinsic group of $\mathbf{K}$ (or that of the dual Kac algebra $\widehat{\mathbf{K}}$) completely determines the structure of the given algebra $\mathbf{K}$ (see [Ta]). So it is one of the important things in the theory of Kac algebras to know the intrinsic group, once one is given a Kac algebra, although it is known (see [DeC1]) that the intrinsic group is not a complete invariant for general Kac algebras. In this direction, De Canière's work [DeC1] should be noted as one of the significant achievements.