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EQUIVARIANT NIELSEN NUMBERS [1]

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In this paper, we introduce equivariant Nielsen type numbers which estimate the minimal number of fixed orbits and fixed points of a G-map $f: X \to X$ in the G-homotopy class of f. As an application, we relate the equivariant Nielsen theory to the Nielsen theory for iterates of maps.

1. Introduction. Let $f: X \to X$ be a self map of an ENR X so that the set of fixed points $\operatorname{Fix} f$ is compact. In topological fixed point theory, the fixed point index I_f ([**D2**]) is an algebraic count of the number of fixed points of f so that $I_f \neq 0$ implies $\operatorname{Fix} f \neq \varnothing$. When X is compact, this algebraic count is given by the Lefschetz number $L(f)$ which can be expressed as a trace. However $L(f)$ does not usually give much information about the size of $\operatorname{Fix} f$. A more subtle invariant $N(f)$, namely the Nielsen number of f, gives a lower bound for the minimal number of fixed points of maps in the homotopy class of f. In many situations, $N(f)$ is a sharp lower bound (e.g. when M is a compact connected manifold of dimension ≥ 3).

Fixed point theory can be generalized to the study of periodic points, i.e., fixed points of $f^n = f \circ \cdots \circ f$. In [**D3**], Dold established a combinatorial relation among the fixed point indices of iterates of f. It was shown by Komiya [**K**] that Dold's relation can be derived from a similar congruence relation on the fixed point indices of equivariant maps. Nielsen fixed point theory was generalized to iterates of maps by Jiang [**J**], Heath-Piccinini-You [**HPY**], Heath-You [**HY**] and some earlier work of Halpern. The objective of this paper is to develop an equivariant Nielsen theory for G-maps. As an application, we indicate how the equivariant theory generalizes the periodic point theory in a similar fashion as [**K**] extends [**D3**].

For background in Nielsen fixed point theory, we refer the reader to [**Br**] and [**J**]. For equivariant topology, the basic references are [**B**] and [**tD**]. This paper is organized as follows.

[1] The results in this paper were announced at the Special Session on Nielsen Fixed Point Theory in the Pre-Congress Topology Conference held at the University of Hawaii, August 12–18, 1990.