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THE FLOW SPACE OF A DIRECTED G-GRAPH

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This paper deals with operator-algebraic aspects of the theory of infinite, locally finite directed graphs. A (complex-valued) function on the set of edges of a directed graph whose sum over the edges pointing out of each vertex equals the sum over the edges pointing in is called a flow. Of particular interest here is the projection of the Hilbert space of square-summable functions on the edges to the closed subspace consisting of the square-summable flows. The flow space projection can be identified in a meaningful and interesting way whenever a group G acts properly on the graph, with the latter finite modulo the action of G and connected. In general, a choice of vertex and edge orbit representatives gives a realization of the flow space projection in an algebra of matrices over the von Neumann algebra of G. Suppressing the dependence on the choice of orbit representatives yields a class in K_0 of this von Neumann algebra. This K_0-class is the sum of the classes arising from the stabilizers of a representative set of edges minus a corresponding sum for vertices. Furthermore, if G is non-amenable, all of the foregoing takes place within the reduced C^*-algebra of G rather than just in the group von Neumann algebra.

1. Preliminaries. We will largely follow the notation and terminology of the first chapter of [4] for directed graphs and group actions. A directed graph X consists of a set V of vertices, a set E of edges, and maps $i, t\colon E \to V$. The edge y joins the initial vertex $i(y)$ to the terminal vertex $t(y)$. We will assume that $(i, t)\colon E \to V \times V$ is injective with range missing the diagonal, i.e. that X has no loops or multiple edges. For a vertex v, we write $\operatorname{star}(v) = i^{-1}(v) \cup t^{-1}(v)$, the set of edges incident at v, and $N(v) = t(i^{-1}(v)) \cup i(t^{-1}(v))$, the set of vertices joined to v by an edge. The cardinality of $\operatorname{star}(v)$ is called the degree of v; we abbreviate $\deg(v) = |\operatorname{star}(v)|$. We will always require X to be locally finite of bounded degree, meaning that $\sup\{\deg(v)\colon v \in V\}$ (which we denote by $\deg(X)$) is finite. (This will ensure that the various Hilbert space operators considered below are all bounded.) A path p in X of length n is a sequence $v_1, y_1, \dots, v_n, y_n, v_{n+1}$, where for $j = 1, \dots, n$, the edge y_j joins the vertices v_j and v_{j+1}. We think of p as having a direction of traverse, from v_1 to v_{n+1}, so each edge y_j will point either forward or backward along p; we set $\langle p, y_j\rangle = 1$ or -1 depending on whether