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GENERAL KAC-MOODY ALGEBRAS AND THE KAZHDAN-LUSZTIG CONJECTURE

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Let $\mathfrak{g}$ be a Kac-Moody algebra defined by a not necessarily symmetrizable generalized Cartan matrix. We use operators of coherent continuation to define modules $U_\alpha L(w \cdot \lambda)$ with α a simple root of $\mathfrak{g}$ and w in the Weyl group W of $\mathfrak{g}$, and then use these modules to study the integers $\dim \operatorname{Ext}^n(M(x \cdot \lambda), L(y \cdot \lambda))$ for x and y in W, where λ is a dominant integral weight, $M(\mu)$ denotes the Verma module over $\mathfrak{g}$ of highest weight μ and $L(\mu)$ denotes its irreducible quotient. In particular, we show that in the presence of a parity conjecture and a weak assumption on the behavior of the modules $U_\alpha L(w \cdot \lambda)$, both of which hold in the case of a finite dimensional $\mathfrak{g}$, we may compute the dimensions by induction on the length of x, recovering the coefficients of "twisted" versions of the Kazhdan-Lusztig polynomials, where the twist comes from the fact that we start at the top of the orbit $W \cdot \lambda$, rather than at the bottom.

1. Introduction. Let $\mathfrak{g}$ be a Kac-Moody algebra over a field K of characteristic zero defined by a not necessarily symmetrizable generalized Cartan matrix. Translation functors over $\mathfrak{g}$ were introduced in [6] and [7]. In the latter, operators of coherent continuation were introduced, but did not possess the desired self-adjointness which is so useful in the finite dimensional case, i.e. where $\mathfrak{g}$ is finite dimensional.

In the present work, we modify the definition of the translation functors somewhat, so that when we compose two translation functors to obtain an operator of coherent continuation, we do not begin and end in the same Weyl group orbit. We do, however, obtain two operators of coherent continuation which are adjoint to each other. We then use these operators of coherent continuation to define modules $U_\alpha L(w \cdot \lambda)$ with α a simple root of $\mathfrak{g}$ and w in the Weyl group W of $\mathfrak{g}$, and use these modules to study the integers $\dim \operatorname{Ext}^n_{C(\lambda)}(M(x \cdot \lambda), L(y \cdot \lambda))$ for x and y in W, where λ is a dominant integral weight, $C(\lambda)$ denotes the category of weight modules all of whose weights are less than or equal to λ, $M(\mu)$ denotes the Verma module over $\mathfrak{g}$ of highest weight μ and $L(\mu)$ denotes its irreducible quotient.

In our study of these dimensions, we show that the various