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# THE STANDARD DOUBLE SOAP BUBBLE IN $\mathbf{R}^2$ UNIQUELY MINIMIZES PERIMETER

JOEL FOISY, MANUAL ALFARO, JEFFREY BROCK,
NICKELOUS HODGES AND JASON ZIMBA

**Of course the circle is the least-perimeter way to enclose a region of prescribed area in the plane. This paper proves that a certain standard "double bubble" is the least-perimeter way to enclose and separate two regions of prescribed areas. The solution for three regions remains conjectural.**

**1. Introduction.** Soap bubbles naturally tend to minimize surface area for given volumes. This paper considers the two-dimensional analog of soap bubbles, seeking the way to fence in prescribed areas using the least amount of perimeter. For one prescribed quantity of area, the answer is of course a circle. This paper shows that for two prescribed quantities of area, the unique answer is the "standard double bubble" of Figure 1.0.1, and not the non-standard competitors admitted by the general existence theory ([Al], [M]) with disconnected bubbles or exterior (see Figure 1.0.2).


FIGURE 1.0.1. The standard double-bubble is the unique least-perimeter way to enclose and separate two prescribed areas.

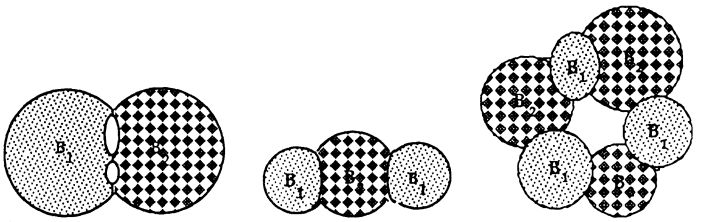


FIGURE 1.0.2. Some non-standard double-bubbles. (A) has connected bubbles but the exterior is disconnected. (B) has a connected exterior, but its $B_1$ bubble is disconnected. (C) has both disconnected bubbles and a disconnected exterior.