

## AN APPLICATION OF THE VERY WEAK BERNOULLI CONDITION FOR AMENABLE GROUPS

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**In this paper, we give an application of the recently developed Very Weak Bernoulli condition for amenable groups. The setting for the application is an attractive particle system with the usual lattice replaced by a general countable amenable group.**

**1. Introduction.** We consider probability measures on  $X = \{0, 1\}^G$  which are invariant under the natural right action of the group  $G$ . Two such measures  $\mu$  and  $\nu$  are *isomorphic* if there exists an  $f: (\{0, 1\}^G, \mu) \rightarrow (\{0, 1\}^G, \nu)$  which is bijective a.e. and measure-preserving and which commutes with the action of  $G$ . A Bernoulli Shift is a stationary process which is isomorphic to an i.i.d. process. For the case  $G = \mathbb{Z}$ , it was proven by Ornstein [6] that entropy is a complete invariant for Bernoulli Shifts. In this work, a number of important properties of a finite state discrete time ( $G = \mathbb{Z}$ ) stationary process were introduced, namely Finitely Determined (FD), Very Weak Bernoulli (VWB), and Weak Bernoulli (WB). This work together with [10] shows that FD and VWB are equivalent to being a Bernoulli Shift. While these are also all implied by the WB condition, they do not imply it. The fact that a Bernoulli Shift is not necessarily WB might seem strange but is partially explained by the fact that WB is not an isomorphism invariant. An example of such a process is given in [13]. The equivalence of Bernoulli and VWB allowed researchers to prove that a number of concrete systems were in fact Bernoulli Shifts.

After this, further equivalent but useful concepts were introduced, namely that of Thouvenot's extremality [11] and that of  $\varepsilon$ -block independence [12]. Later on, it was natural to extend as much as possible the Bernoulli theory to the group  $\mathbb{Z}^d$  and ultimately to amenable groups. Kieffer [4] succeeded in obtaining a Shannon-McMillan theorem in the amenable group setting. (This was later improved to a pointwise theorem for certain Følner sequences in [8].) Recently, the theorem that entropy is a complete invariant for Bernoulli Shifts was generalized to the case where the group acting is a general countable, discrete, amenable group [7]. (In fact, they handle the broader class