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ON THE UNIFORM APPROXIMATION PROBLEM FOR THE SQUARE OF THE CAUCHY-RIEMANN OPERATOR

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Let X be a compact subset of the plane and f a continuous function on X satisfying the equation $\overline{\partial}^2 f = 0$ in the interior of X. It is unknown whether f can be uniformly approximated on X by functions g satisfying the equation $\overline{\partial}^2 g = 0$ in some neighbourhood (depending on g) of X. We show that this is the case under the additional assumption that f satisfies a Dini-type continuity condition.

1. Introduction. Let L be a constant coefficients elliptic differential operator in $\mathbb{R}^d$. Given a compact $X \subset \mathbb{R}^d$ let $H(X, L)$ be the closure in $C(X)$ of the set

$$\{f|_X : Lf = 0 \text{ on some neighbourhood of } X\}.$$

It is clear that a function in $H(X, L)$ necessarily belongs to

$$h(X, L) = C(X) \cap \{Lf = 0 \text{ on the interior of } X\}.$$

The uniform approximation problem for the operator L consists in characterizing those X for which $H(X, L) = h(X, L)$. Since $h(X, L) = C(X)$ if and only if X is nowhere dense, our problem restricted to nowhere dense compact sets becomes that of describing those X for which $H(X, L) = C(X)$.

A complete solution for $L = \Delta$ (the Laplacian) was independently obtained in the forties by Deny [2] and Keldysh [6] using a duality approach relying on potential theoretic methods. Denoting by Cap the Wiener capacity of classical potential theory, their result can be stated as follows.

THEOREM (*Deny-Keldysh*). *The identity* $H(X, \Delta) = h(X, \Delta)$ *occurs if and only if one has* $\operatorname{Cap}(B \setminus \overset{\circ}{X}) = \operatorname{Cap}(B \setminus X)$ *for each open ball* B.

Vitushkin [11] solved in the sixties the problem for $L = \overline{\partial}$ (the Cauchy-Riemann operator in the plane) introducing his far reaching