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THE PLANCHEREL FORMULA FOR HOMOGENEOUS SPACES WITH POLYNOMIAL SPECTRUM

RONALD L. LIPSMAN

The distribution-theoretic version of the Plancherel formula—known as the Penney-Fujiwara Plancherel Formula—for the decomposition of the quasi-regular representation of a Lie group G on $L^2(G/H)$ is considered. Attention is focused on the case that the spectrum consists of irreducible representations induced from a finite-dimensional representation. This happens with great regularity for *Strichartz* homogeneous spaces wherein G and H are semidirect products of normal abelian subgroups by a reductive Lie group. The results take an especially simple form if G/H is symmetric. Criteria for finite multiplicity and for multiplicity-free spectrum are developed. In the case that G is a motion group—the original situation stressed by Strichartz—the results are particularly striking.

0. Introduction. This paper is a sequel to and generalization of [7]. In that paper we derived the Penney-Fujiwara version of the Plancherel formula for the quasi-regular representation of a homogeneous space with monomial spectrum. That is, for G a (connected) Lie group, $H \subset G$ a closed subgroup, and $\tau = \mathrm{Ind}_H^G 1$ with the property that a.a. of the irreducibles that appear in the spectrum of τ are induced from a *character*, we derived a distribution-theoretic version of the Plancherel formula for τ. We remark that (as observed in [7] or [6]) such a formula gives us the explicit intertwining operator for the direct integral decomposition of τ, as well as a determination of the Plancherel measure. A specific situation to which the results of [7] apply is that of abelian symmetric spaces (defined originally in [5]).

The nature of the generalization of [7] in this paper is two-fold. First we will replace the hypothesis of monomial spectrum by that of polynomial spectrum—meaning that a.a. the representations in the spectrum of τ are induced from *finite-dimensional* representations. Second we shall expand the abelian symmetric space application to a much broader family of symmetric spaces introduced and studied by Strichartz. Regarding the first extension, it is quite a natural step to take. Approaching either the minimal principal series of a semisimple Lie group or the generic representations of a type I amenable group,