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SKEINS AND HANDLEBODIES

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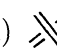
The Temperley-Lieb algebra is used to find a base for the vector space that is associated to a closed surface by the Topological Quantum Field Theory corresponding to the original Jones polynomial invariant.

1. Introduction. The Kauffman linear skein $\mathbb{S}M$ of an oriented 3-manifold M, that has a (possibly empty) finite collection of (framed) points specified in its boundary, is defined as follows. Throughout, A will be a fixed complex number later to be chosen to be a primitive $4r$th root of unity (though it is equally possible to work with the ring of Laurent polynomials in A, quotiented by the ideal generated by a cyclotomic polynomial). $\mathbb{S}M$ is the vector space of formal linear sums of isotopy classes of framed links in M of disjoint simple closed curves and arcs that agree with the specification in ∂M, quotiented by the following relations.

(i)  $= A$  $+ A^{-1}$ 

(ii) $L \cup U = (-A^{-2} - A^{2})L$.

In (i) a framing on a curve is depicted by a parallel to the curve, and in (ii) U is a zero-framed unknotted component in a ball disjoint from the link L. It is often convenient to project M to some surface F (for example, S^3 less two points projects to S^2, a handlebody projects to a disc-with-holes) and then $\mathbb{S}M$ is interpreted as the linear skein $\mathbb{S}F$ of link diagrams in F as in [5], [7], [10], the framings being determined by parallel curves in F. In particular the nth Temperley-Lieb algebra is the Kauffman skein of the ball with two sets of n points specified on its boundary. It is convenient to consider that via link diagrams in a rectangle with n specified points on the left edge, n points on the right edge, the product in the algebra coming from juxtaposition of the rectangles. Now, it is clear that $\mathbb{S}S^3 = \mathbb{C}$; in fact a zero-framed link corresponds in $\mathbb{S}S^3$ to its Jones polynomial evaluated at $t = A^{-4}$. Suppose that M is embedded in S^3, that M' is the closure of $S^3 - M$ and that M and M' have the same specified framed points in their