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DIFFERENTIAL-DIFFERENCE OPERATORS AND MONODROMY REPRESENTATIONS OF HECKE ALGEBRAS

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Associated to any finite reflection group G on an Euclidean space there is a parametrized commutative algebra of differential-difference operators with as many parameters as there are conjugacy classes of reflections. The Hecke algebra of the group can be represented by monodromy action on the space of functions annihilated by each differential-difference operator in the algebra. For each irreducible representation of G the differential-difference equations lead to a linear system of first-order meromorphic differential equations corresponding to an integrable connection over the G-orbits of regular points in the complexification of the Euclidean space. The fundamental group is the generalized Artin braid group belonging to G, and its monodromy representation factors over the Hecke algebra of G. Monodromy has long been of importance in the study of special functions of several variables, for example, the hyperlogarithms of Lappo-Danilevsky are used to express the flat sections and the work of Riemann on the monodromy of the hypergeometric equation is applied to the case of dihedral groups.

Orthogonal polynomials and special functions of classical type in several variables arise from analysis on root systems. Generally there is an underlying definite integral with a number of parameters. To evaluate such an integral in closed form means to obtain a formula in terms of known special functions, especially the gamma function. These formulas are generally meromorphic and allow analytic continuation of the parameter values into regions where the integral is no longer defined. In order to understand the singularities one is led to deep problems in Coxeter and Artin groups, Hecke algebras, their representations, and differential equations. Certain polynomials in a parameter such as Poincaré series and generic degrees of representations are associated to such groups. The logarithms of their zero-sets are closely related to the aforementioned integrals.

In previous work [**Du1**, **3**] mainly concerned with orthogonal polynomials associated to finite reflection groups the author constructed a commutative algebra of differential-difference operators for such groups, with as many parameters as there are conjugacy classes of