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SOLUTIONS OF THE STATIONARY AND NONSTATIONARY NAVIER-STOKES EQUATIONS IN EXTERIOR DOMAINS

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It is shown that a nonstationary exterior Navier-Stokes flow tends to a small stationary flow in L^2 like $t^{-3/4}$ as $t \to \infty$.

0. Introduction. In this paper we are concerned with the stationary Navier-Stokes equations

$$(0.1)\qquad \begin{aligned} &(w \cdot D)w - \Delta w + D\overline{p} = f, \quad D \cdot w = 0 \quad \text{in } G, \\ &w = 0 \quad \text{on } \partial G \quad (D = \text{grad}), \end{aligned}$$

and the nonstationary Navier-Stokes equations

$$\begin{aligned} v_t + (v \cdot D)v - \Delta v + D\overline{\overline{p}} = f \quad &\text{in } G \times (0, \infty), \\ D \cdot v = 0 \quad &\text{in } G \times (0, \infty), \\ v = 0 \quad &\text{on } \partial G \times (0, \infty), \\ v|_{t=0} = a + w \quad &\text{in } G \quad (v_t = \partial v / \partial t). \end{aligned}$$

Here and in what follows G denotes a smooth exterior domain of R^3, $f = f(x)$ is a prescribed vector field, and $\overline{p}$ (resp. $\overline{\overline{p}}$) represents unknown stationary (resp. nonstationary) scalar pressure which can be determined by the stationary solution w via (0.1) (resp. nonstationary solution v via (0.2)).

As is well known, it was shown by Finn [**8**, **9**] that (0.1) admits a small solution

$$(0.3)\qquad \begin{aligned} &w \in L^\infty(G; R^3), \qquad Dw \in L^3(G; R^9), \\ &C_0 = \sup_{x \in G} |x|\,|w(x)| < \infty. \end{aligned}$$

If $C_0 < 1/2$ the Finn's solution w may be formed as a limit of a nonstationary solution v as $t \to \infty$ in local or global L^2-norms (cf. Heywood [**15**, **14**], Galdi and Rionero [**11**], Miyakawa and Sohr [**23**]) and in other norms (cf. Heywood [**16**], Masuda [**20**]). Moreover it has recently proved (cf. Borchers and Miyakawa [**4**]) that every weak solution of (0.2) tends the Finn's solution in $L^2(G; R^3)$