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L^p-INTEGRABILITY OF THE SECOND ORDER DERIVATIVES OF GREEN POTENTIALS IN CONVEX DOMAINS

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We give estimates in L^p, $1 < p \leq 2$, of the second order derivatives of the Green potential of $f \in L^p$, for convex domains. This is done by interpolating between estimates in L^1 and L^2 of functions in atomic H^1 and L^2, respectively. The crucial step is obtaining the atomic estimate which is done by adapting to the present situation, a technique introduced by Dahlberg and Kenig.

0. Introduction. The purpose of this paper is to prove that for a convex domain Ω in $\mathbf{R}^n$, the second order derivatives of the Green potential of f, $\nabla_2 Gf$, are in $L^p(\Omega)$ if $f \in L^p(\Omega)$ with $1 < p \leq 2$. To avoid technicalities we assume throughout the paper that $n \geq 3$. The main results of the paper are

THEOREM 1. *Suppose D is a convex domain above a Lipschitz graph in $\mathbf{R}^n$, i.e., $D = \{x_n > \varphi(x')\}$ where φ is a convex Lipschitz function with Lipschitz constant bounded by M and $x' \in \mathbf{R}^{n-1}$. Let G be the Green function for D and let the Green potential for $f \in L^p(D)$, $1 < p \leq 2$, be denoted by Gf. Then we have that*

$$\int_D |\nabla_2 Gf|^p \, dx \leq c \int_D |f|^p \, dx,$$

where ∇_2 denotes the second order derivatives and the constant c only depends on the Lipschitz constant M. □

Via a patching argument we will derive the case of a bounded convex domain from the results of Theorem 1.

THEOREM 2. *Let Ω be an open, bounded and convex domain in $\mathbf{R}^n$, $n \geq 3$. Let G be its Green function. Suppose $1 < p \leq 2$. Then $\nabla_2 Gf \in L^p(\Omega)$ and*

$$\int_\Omega |\nabla_2 Gf|^p dx \leq C \int_\Omega |f|^p dx,$$