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SINGULAR HOMOLOGY AND COHOMOLOGY WITH LOCAL COEFFICIENTS AND DUALITY FOR MANIFOLDS

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This article contains an application of the author's previous work on cohomology theories on a space to an exposition of singular theory. After a summary of the relevant concepts concerning cohomology theories in general, singular homology and singular cohomology with local coefficients are defined. Each of these is presented in two versions, one with compact supports and one with arbitrary closed supports. It is shown that each version satisfies an appropriate duality theorem for arbitrary (i.e. nonorientable) topological manifolds.

1. Introduction. This paper is a presentation of singular homology and cohomology theory with local coefficients. Included is a treatment of the usual singular homology with compact supports (which is based on finite chains) and the singular homology based on locally finite chains. The former is a weakly additive theory and the latter is an additive theory.

Similarly, there are two types of singular cohomology, one with compact supports and one with arbitrary supports. In an n-manifold X the basic duality theorem asserts the isomorphism of the two types of q-dimensional homology for an open pair (U, V) in X to the corresponding two types of $(n-q)$-dimensional cohomology of the complementary closed pair $(X-V, X-U)$ with coefficient systems suitably related.

Our approach is to study homology and cohomology on a fixed space X and to prove the duality theorem referred to above by comparing two cohomology theories on X, one being the appropriate homology of the open pair in complementary dimension and the other being the corresponding cohomology theory of the complementary closed pair. For this we present the relevant concepts concerning such theories and a review of the comparison theorem for them.

Thus, the paper is divided into two parts, §§2 through 5 are devoted to general concepts concerning covariant and contravariant functors defined on pairs in a space, and §§6 through 10 are devoted to applications of these ideas to singular homology and cohomology and to a proof of the duality theorem for manifolds.