

MAPS BETWEEN SEIFERT FIBERED SPACES OF INFINITE π_1

YONGWU RONG

A theorem of A. Edmonds says that any nonzero degree map between closed surfaces is homotopic to a composition of a pinch map and a branched covering. Here we consider the analogous problem in dimension three. We prove that any nonzero degree map between P^2 -irreducible Seifert fibered spaces of infinite π_1 is homotopic to a composition of “vertical pinches” and a fiber preserving branched covering, except for a few cases which we describe completely. In particular, any such degree one map is homotopic to a composition of vertical pinches.

0. Introduction. In this paper we study nonzero degree maps between closed P^2 -irreducible Seifert fibered spaces of infinite π_1 , or equivalently, closed aspherical Seifert fibered spaces. We prove that any such map is homotopic to a composition of vertical pinches (defined in §1) and a fiber preserving branched covering, except for certain cases which can be completely understood (Theorem 3.2). As a corollary, any degree one map between such spaces is homotopic to a composition of vertical pinches.

The analogous theorem for surfaces was proved by A. Edmonds [1]. Later R. Skora gave a simplified proof using the notion of geometric degree [6]. Our proof uses similar ideas as theirs. Some extra work must be done to adjust the map so that it is nice with respect to the Seifert fibrations of the manifolds.

In §1 we establish terminology. Pinches and squeezes are defined by analogy with those definitions in dimension two given by A. Edmonds [1]. In §2 we show our map can be homotoped into an equivariant fiber preserving map, but possibly followed by a covering between Euclidean manifolds (those which have the geometry of E^3). In §3 we give an inductive proof of our main theorem.

I would like to thank R. Fintushel for helpful conversations, and R. Skora for his useful comment.

1. Notations and terminology. For a Seifert fibered space M , h denotes either a regular fiber or its homotopy class. Tori and annuli are often regarded as Seifert fibered without singular fibers, and h has