

A HOMOTOPY TRANSFER FOR FINITE GROUP ACTIONS

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We obtain a transfer for group actions on spaces for which the orbit map admits a section. This transfer exists for sets of homotopy classes as well as for any generalized homology theory.

Introduction. Intuitively, one feels that there should exist special relationships between the homotopy invariants of a space Y and its quotient by the action of some finite group G . The main result of this paper is the construction, for an arbitrary homology theory, of a version of the transfer that exists for ordinary homology. Recall that this is a homomorphism $\tau: H_n(Y/G) \rightarrow H_n(Y)$ with the following properties:

- (a) $\tau \circ \rho(z) = \sum_{g \in G} g * z$ for all $z \in H_n(Y)$,
- (b) $\rho \circ \tau(v) = |G|v$ for all $v \in H_n(Y/G)$

where $\rho: H_n(Y) \rightarrow H_n(Y/G)$ is the projection. An account of this can be found in [Br].

Unfortunately, the existence of a transfer map satisfying (a) and (b), or their duals in cohomology, seems to be a special property of the ordinary homology and cohomology functors which is closely tied to the fact that Eilenberg-Mac Lane spaces have the homotopy type of abelian monoids. In view of this, it is not surprising that in general there is no transfer for covariant functors F such as π_n and those associated with generalized homology theories.

In this paper we will recover a version of transfer for many functors F including generalized homology theories. In order to deal with the fact that the H -spaces that arise are not, in general, of the homotopy type of abelian monoids, we will have to multiply equation (a) by a number $c(G)$, that I have been calling the coherence number of the group G . This number depends only on the group G and is currently under intense investigation. For cyclic groups this number is 1 and hence the transfer equations will have their usual form in this case. It is not yet known whether this number is always finite, so there may be groups to which our transfer cannot be applied, although our feeling