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ON DISCRETE ISOMETRY GROUPS OF NEGATIVE CURVATURE

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In this paper we extend well-known results concerning the algebraic limits and deformations of groups of hyperbolic isometries of hyperbolic 3-space, H^3, to *negatively curved groups*. For us these will be groups of isometries of variable negative curvature metrics satisfying a pinching condition and in particular will include the $\mathbb{R}$-rank one Lie groups. We accomplish these goals, as in the hyperbolic case, by producing a version of Jørgensen's inequality for such groups. Using an appropriate normalisation we can consider algebraic limits and deformations of such groups in the homeomorphism group of the n-ball, $\mathrm{Hom}(\mathbf{B}^n)$. We ask that the generators of each group move continuously or some sequence of generators have limits in $\mathrm{Hom}(\mathbf{B}^n)$, but there is no such restriction on the associated negatively curved metrics. We then recover many of the standard results for groups of hyperbolic isometries of H^3 in this more general setting under mild and usually necessary restrictions, such things as the limits being discrete, or the deformations are algebraically trivial and so forth.

We point out, as a warning, that some authors use the term negatively curved groups to mean hyperbolic groups in the sense of Gromov [G].

The pinching condition we assume may be relaxed if the curvature is nonpositive, bounded below and the associated Hadamard manifold is a visibility manifold [EO]. The version of Jørgensen's inequality [J] that we produce follows more or less directly as in [M], where the n-dimensional hyperbolic case is considered, from Gromov's generalization of the convergence of iterated commutators, see Buser and Karcher [BK, §2]. The results of this paper can be viewed as further applications of that fundamental result. Other aspects of the theory of isometry groups of negative curvature can be found in the book of Ballman, Gromov and Schroeder [BGS] which we use as a general reference.

In §§3, 4 we discuss the algebraic limits of negatively curved groups (in the appropriate homeomorphism group) and show that under mild (and necessary) hypotheses they are discrete. We also obtain that a group of isometries of a negatively curved metric is discrete if and only