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THE MODULI OF RATIONAL WEIERSTRASS FIBRATIONS OVER $\mathbf{P}^1$: SINGULARITIES

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The Weierstrass equation $y^2 = x^3 + ax + b$, where a and b are rational functions of one variable, defines a fibration over $\mathbf{P}^1$, which we call a Weierstrass fibration. We consider the moduli space W of rational Weierstrass fibrations over $\mathbf{P}^1$. In this paper we determine the singular locus of W and we compute the general singularities. We work over $\mathbf{C}$, but it seems possible to generalize our methods to characteristic $p \neq 2, 3$.

Introduction. In [**Mi**] Miranda has constructed moduli spaces W_N, $N \geq 0$, for Weierstrass fibrations over $\mathbf{P}^1$ whose zero section has self intersection number $-N$ in the associated elliptic surface. Seiler has generalized and extended this work in [**Sei2**] and [**Sei3**]. For $N = 1$, we have the moduli space of rational fibrations $W = W_1$. The points of W parametrize isomorphism classes of rational Weierstrass fibrations over $\mathbf{P}^1$ with at most rational double point singularities whose associated elliptic surface (= minimal resolution of singularities) has only reduced fibers. By passing to the associated elliptic surface, W can be viewed as parametrizing isomorphism classes of relatively minimal elliptic surfaces over $\mathbf{P}^1$ admitting a section which have only reduced fibers. The basic definitions and constructions are reviewed in §1.

To determine the singular locus of W, we first find the locus S of Weierstrass fibrations that have non-negligible (= nontrivial) automorphisms. By means of the Weierstrass equation, this boils down to finding stable pairs of Weierstrass coefficients whose isotropy group with respect to the action of $G = \mathbf{GL}_2/\pm I$ is nontrivial. This work is the content of §2 and culminates in Theorem 1 where the 7 irreducible components of S are listed.

The general singularities turn out to be cyclic quotient singularities. We compute and classify them with the help of the slice theorem and work of Prill [**Pr**] in Theorem 2, §3.

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