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FIXED POINTS OF SURFACE DIFFEOMORPHISMS

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We give a complete proof of the following theorem which was conjectured by Jakob Nielsen for closed oriented surfaces.

THEOREM. ***Let $f\colon M \to M$ be a homeomorphism of a compact surface. When M is closed, then f is isotopic to a diffeomorphism with $N(f)$ fixed points, where $N(f)$ is its Nielsen number. When M has boundary, $N(f)$ should be replaced by the relative Nielsen number $N(f; M, \partial M)$ defined by Schirmer.***

Another result is the inequality $|L(f) - \chi(M)| \leq N(f) - \chi(M)$ when $\chi(M) < 0$, where $L(f)$ is the Lefschetz number and $\chi(M)$ is the Euler characteristic.

Introduction. For a self-map f of a compact polyhedron X, the Nielsen number $N(f)$ is defined to be the number of essential fixed point classes. (See **[J3]** for an introduction to the Nielsen fixed point theory.) It is a classical theorem of Wecken **[W]** that $N(f)$ is a lower bound of the number of fixed points for all maps homotopic to f, and that if X is a manifold of dimension ≥ 3, this lower bound is always realizable (see also **[Br]**, **[K]**). It is now known **[J4]** that when X is a surface with negative Euler characteristic, there exists a map $f\colon X \to X$ such that every map homotopic to f has more than $N(f)$ fixed points. The purpose of this paper is to show that for homeomorphisms of surfaces the Nielsen number is indeed the least number of fixed points in the isotopy class, as Nielsen himself conjectured (cf. **[N2**, §31**]**) in his study of oriented closed surfaces.

MAIN THEOREM. *Let M be a compact surface, closed or with boundary. Let $f\colon M \to M$ be a homeomorphism. Then f is isotopic to a smooth embedding which has $N(f)$ fixed points. If, in addition, no boundary component of M is mapped onto itself by f in an orientation-reversing manner, then f is isotopic to a diffeomorphism having $N(f)$ fixed points.*

This theorem was announced in **[J2]**, here strengthened with smoothness considerations. An example in **[J1]** shows it is necessary