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ON THE SHAPE OF FUNDAMENTAL DOMAINS IN $\mathrm{GL}(n, \mathbf{R})/\mathrm{O}(n)$

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We investigate parameters for the symmetric space $H = G/K$, $G = \mathrm{GL}(n, \mathbf{R})$, $K = \mathrm{O}(n)$, in the sense of positive definite quadratic forms. This leads to a description for the fundamental domain H/Γ where Γ is an arithmetic subgroup of G. We also see interesting relations with the Siegel sets. This enables us to explicitly describe Satake compactifcations of H/Γ. We will also consider the behavior at the "bottom" of the fundamental domains.

1. Introduction. The problem of reduction of quadratic forms is an old one. When the subject is positive definite quadratic forms, the first definition of reduction was achieved by Hermite [**8**]. However, it is Minkowski's reduction that is the most familiar, primarily because when we view the quadratic forms geometrically, Minkowski's reduction domain is easily seen to have a finite number of boundary components while this was not known for Hermite's. One may consult Cassels [**3**] or Terras [**14**] for more of the historical details and for a definition of these domains.

In modern language, positive definite quadratic forms may be considered as a symmetric space. Denote the space of positive quadratic forms by $\mathscr{P}_n$, and let $G = \mathrm{GL}(n, \mathbf{R})$, $K = \mathrm{O}(n)$. Then $\mathscr{P}_n$ may be identified with G/K by:

$$gK \mapsto {}^T g g$$

for any $g \in G$. We will be most interested in the space of quadratic forms of determinant one which will be denoted $\mathscr{S}\mathscr{P}_n$. This may be obtained from G/K by modding out by the center of G. If Γ_n stands for $\mathrm{GL}(n, \mathbf{Z})/\{\pm I\}$, Γ_n acts discontinuously on $\mathscr{S}\mathscr{P}_n$ by:

$$Z \mapsto Z[\gamma] = {}^T\gamma Z \gamma$$

where $Z \in \mathscr{S}\mathscr{P}_n$ and $\gamma \in \Gamma_n$. We will always use this notation $Z[X] = {}^T X Z X$ where $X \in \mathbf{R}^{n \times k}$ for any k (including $k = 1$ so that X is a vector). Then a fundamental domain in $\mathscr{S}\mathscr{P}_n$ under the action of Γ_n is a subset of $\mathscr{S}\mathscr{P}_n$ which may be identified with the quotient space $\mathscr{S}\mathscr{P}_n/\Gamma_n$ and which represents the reduced forms. For