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LACUNARY STATISTICAL CONVERGENCE

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The sequence x is statistically convergent to L provided that for each $\varepsilon > 0$,

$$\lim_n n^{-1}\{\text{the number of } k \le n\colon |x_k - L| \ge \varepsilon\} = 0.$$

In this paper we study a related concept of convergence in which the set $\{k\colon k \le n\}$ is replaced by $\{k\colon k_{r-1} < k \le k_r\}$, for some lacunary sequence $\{k_r\}$. The resulting summability method is compared to statistical convergence and other summability methods, and questions of uniqueness of the limit value are considered.

1. Introduction. A complex number sequence x is said to be *statistically convergent* to the number L if for every $\varepsilon > 0$,

$$\lim_n \frac{1}{n}|\{k \le n\colon |x_k - LK| \ge \varepsilon\}| = 0, \tag{1}$$

where the vertical bars indicate the number of elements in the enclosed set. In this case we write $S\text{-}\lim x = L$ or $x_k \to L(S)$. We shall also use S to denote the set of all statistically convergent sequences. The idea of statistical convergence was introduced by Fast [4] and studied by several authors [2], [3], [5], [6], [11]. There is a natural relationship [2] between statistical convergence and strong Cesàro summability:

$$|\sigma_1| := \left\{x\colon \text{ for some } L,\ \lim_n \left(\frac{1}{n}\sum_{k=1}^{n}|x_k - L|\right) = 0\right\}.$$

By a *lacunary sequence* we mean an increasing integer sequence $\theta = \{k_r\}$ such that $k_0 = 0$ and $h_r := k_r - k_{r-1} \to \infty$ as $r \to \infty$. Throughout this paper the intervals determined by θ will be denoted by $I_r := (k_{r-1}, k_r]$, and the ratio k_r/k_{r-1} will be abbreviated by q_r. There is a strong connection [7] between $|\sigma_1|$ and the sequence space N_θ, which is defined by

$$N_\theta := \left\{x\colon \text{for some } L,\ \lim_r \left(\frac{1}{h_r}\sum_{k\in I_r}|x_k - L|\right) = 0\right\}.$$

The purpose of this paper is to introduce and study a concept of convergence that is related to statistical convergence (1) in the same way that N_θ is related to $|\sigma_1|$.