PACIFIC JOURNAL OF MATHEMATICS
Vol. 160, No. 1, 1993

BOURGAIN ALGEBRAS ON THE UNIT DISK

JOSEPH A. CIMA, KAREL STROETHOFF AND KEITH YALE

The Bourgain algebra of $H^\infty(\mathbb{D})$ relative to $L^\infty(\mathbb{D})$ is shown to be $H^\infty(\mathbb{D}) + C(\overline{\mathbb{D}}) + V$, where V is an ideal of functions in $L^\infty(\mathbb{D})$ which vanish in an appropriate sense near the boundary of $\mathbb{D}$.

1. Introduction. Let $\mathscr{X}$ be a commutative Banach algebra with an identity and let $\mathscr{A}$ be a linear subspace of $\mathscr{X}$. J. Cima and R. Timoney **[6]** introduced the notion of the Bourgain algebra based on ideas of J. Bourgain [3]: the *Bourgain algebra* $\mathscr{A}_b$ consists of those f in $\mathscr{X}$ such that

(1) if $f_n \to 0$ weakly in $\mathscr{A}$, then $\operatorname{dist}(f_n f, \mathscr{A}) \to 0$.

The distance $\operatorname{dist}(f_n f, \mathscr{A})$ between $f_n f$ and $\mathscr{A}$ is the quotient norm of the coset $f_n f + \mathscr{A}$ in the space $\mathscr{X}/\mathscr{A}$. The proof in **[6]** shows that $\mathscr{A}_b$ is a closed subalgebra of $\mathscr{X}$ and if $\mathscr{A}$ is an algebra then $\mathscr{A} \subseteq \mathscr{A}_b$. It is important to note that $\mathscr{A}_b$ depends upon the space $\mathscr{X}$ even though this is not reflected in the notation. For a brief survey of Bourgain algebras see K. Yale **[16]**.

Let $H^\infty(\mathbb{D})$ be the algebra of bounded analytic functions on the open unit disk $\mathbb{D}$. There are at least three different natural spaces $\mathscr{X}$ containing $H^\infty(\mathbb{D})$. First we can let $\mathbb{T} = \partial\mathbb{D} = \{z \in \mathbb{C} : |z| = 1\}$ be the unit circle and consider the algebra $H^\infty(\mathbb{T})$ of boundary values of $H^\infty(\mathbb{D})$ functions as a subalgebra of $\mathscr{X} = L^\infty(\mathbb{T})$. In this context, J. Cima, S. Janson and K. Yale **[5]** showed that $H^\infty(\mathbb{T})_b = H^\infty(\mathbb{T}) + C(\mathbb{T})$. Another setting is to use the Gelfand map and regard $H^\infty(\mathbb{D})$ as a subalgebra of $\mathscr{X} = C(\mathscr{M})$, where $\mathscr{M}$ denotes the maximal ideal space of $H^\infty(\mathbb{D})$. In this context $H^\infty(\mathbb{D})_b$ has been determined by P. Ghatage, S. Sun and D. Zheng **[10]**. Yet another natural setting is to regard $H^\infty(\mathbb{D})$ as a subalgebra of $\mathscr{X} = L^\infty(\mathbb{D})$, where $L^\infty(\mathbb{D})$ is the usual space of equivalence classes of essentially bounded measurable functions on $\mathbb{D}$ with respect to area measure. The purpose of this paper is to determine $H^\infty(\mathbb{D})_b$ in the latter context. There is no Chang-Marshall theory for $L^\infty(\mathbb{D})$ in contrast to the well-known description of subalgebras between $H^\infty(\mathbb{T})$ and $L^\infty(\mathbb{T})$ which was used in **[5]** to determine $H^\infty(\mathbb{T})_b$. For a survey of the