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A NONEXISTENCE RESULT FOR THE n-LAPLACIAN

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Let P be a point in $\mathbb{R}^n$, $n \geq 2$; then the problem $\operatorname{div}(|\nabla u|^{n-2}\nabla u) = e^u$ **with** $u \in W^{1,n}_{\text{loc}} \cap L^\infty_{\text{loc}}$ **has no subsolutions in** $\mathbb{R}^n \backslash \{P\}$.

Introduction. Let $P = P(x_1, x_2, \dots, x_n)$ be a point in $\mathbb{R}^n$, $n \geq 2$, and $\Omega = \mathbb{R}^n \backslash \{P\}$. Without any loss of generality we will take P to be the origin. Consider the problem

$$(1.1) \qquad \begin{cases} L_p u = e^u \quad \text{in } \Omega, \\ u \in W^{1,p}_{\text{loc}}(\Omega) \cap L^\infty_{\text{loc}}(\Omega); \qquad p > 1. \end{cases}$$

Here $L_p u \equiv \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian with $1 < p < \infty$. By a subsolution u of (1.1) we will mean that $u \in W^{1,p}_{\text{loc}}(\Omega) \cap L^\infty_{\text{loc}}(\Omega)$, and

$$\int_\Omega |\nabla u|^{p-2}\nabla u, \ \nabla\psi + \int_\Omega e^u \psi \leq 0, \quad \forall \psi \in C_0^\infty(\Omega) \text{ and } \psi \geq 0.$$

It is known that for $1 < p < n$, (1.1) has no subsolutions in the exterior of a compact set [AW]. However, for $p = n$ there exist radial subsolutions for large values of $|x|$. We show that (1.1) has no subsolutions in Ω, thus extending the results of [AW], namely

THEOREM 1. *The following problem*

$$L_n u = e^u \quad \text{in } \Omega, \ n \geq 2,$$

has no subsolutions in $W^{1,n}_{\text{loc}}(\Omega) \cap L^\infty_{\text{loc}}(\Omega)$.

The proof of Theorem 1 will be a consequence of a comparison principle and nonexistence of global radial solutions. The proof is presented in §4.

2. Preliminary results.

LEMMA 2.1. *Consider*

$$C(x) = \frac{(1+x)^{1/n}}{1+x^{1/n}} \quad \text{in } 0 \leq x \leq 1.$$

Then $C(x)$ *is decreasing on* $[0, 1]$.