PACIFIC JOURNAL OF MATHEMATICS
Vol. 160, No. 1, 1993

INEQUALITIES FOR QUASICONFORMAL MAPPINGS IN SPACE

G. D. ANDERSON, M. K. VAMANAMURTHY AND M. VUORINEN

A new lower bound for the conformal capacity of the Grötzsch ring and sharp bounds for the radial distortion of a quasiconformal automorphism of the unit ball are obtained in n-space, $n \geq 2$.

1. Introduction. The conformal capacities of the Grötzsch and Teichmüller extremal rings in $\mathbb{R}^n$, $n \geq 2$ (see §2), are denoted by

$$\gamma_n(s) = \operatorname{cap} R_{G,n}(s) \quad \text{and} \quad \tau_n(t) = \operatorname{cap} R_{T,n}(t), \tag{1.1}$$

respectively, where $s > 1$ and $t > 0$. The modulus $M_n(r)$ of the Grötzsch ring $R_{G,n}(1/r)$, $0 < r < 1$, is defined by

$$\gamma_n(1/r) = \omega_{n-1} M_n(r)^{1-n}, \tag{1.2}$$

where ω_{n-1} is the $(n-1)$-dimensional measure of the unit sphere S^{n-1} in $\mathbb{R}^n$. The capacities in (1.1) are related [**G**, §18] by

$$\gamma_n(s) = 2^{n-1}\tau_n(s^2-1), \qquad s > 1. \tag{1.3}$$

For $K > 0$ define increasing homeomorphisms $\varphi_{K,n}$ and $\psi_{K,n}$ from $(0, 1)$ onto $(0, 1)$ by

$$\begin{cases} \varphi_{K,n}(r) = 1/\gamma_n^{-1}(K\gamma_n(1/r)) = M_n^{-1}(\alpha M_n(r)), \\ \psi_{K,n}(r) = (1-\varphi_{1/K,n}^2(r'))^{1/2}, \end{cases} \tag{1.4}$$

where $r' = \sqrt{1-r^2}$ and $\alpha = K^{1/(1-n)}$. Given a domain D in $\mathbb{R}^n$, for $K \geq 1$ let $QC_K(D)$ and $QR_K(D)$ denote the class of all K-quasiconformal and K-quasiregular mappings, respectively, of D into itself [**V1**], [**Vu2**]. For $K \geq 1$, $0 < r < 1$, define [**AVV2**]

$$\varphi_{K,n}^*(r) = \sup\{|f(x)| : |x| = r, f(0) = 0, f \in QC_K(B^n)\}, \tag{1.5}$$

$$\varphi_{1/K,n}^*(r) = \inf\{|f(x)| : |x| = r, f(0) = 0, \\ f \in QC_K(B^n), f(B^n) = B^n\}. \tag{1.6}$$

We extend the functions in (1.4), (1.5), (1.6) to [0,1] by defining them to be 0 at 0 and 1 at 1. For $n \geq 2$, $K \geq 1$, $0 < r < 1$, these