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TAU FUNCTIONS FOR THE DIRAC OPERATOR IN THE EUCLIDEAN PLANE

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In this paper, the τ-functions introduced by M. Sato, M. Miwa, and T. Jimbo in their study of monodromy preserving deformations of the Dirac equation are rigorously identified as determinants of singular Dirac operators. The singular Dirac operators have branched functions in their domains that reflect the monodromy in the deformation theory. The principal result is a new formula for the τ-function, obtained by trivializing a suitable determinant bundle, that can be simply related to the deformation theory and which may also be computed in the transfer matrix formalism. These two different ways of understanding the τ-function provide the link between the deformation theory and the quantum field theory significance of τ-function as a correlation function. This connection is the central result of the Sato-Miwa-Jimbo theory of Holonomic Fields.

Introduction. In this paper we develop a new version of the Sato-Miwa-Jimbo theory of τ-functions for the Euclidean Dirac operator acting in the plane $\mathbf{R}^2$. Before attempting to explain the features of this analysis which are new it will be useful to recall the original setting and results in [**14**]. In 1973 Wu, McCoy, Tracy, and Barouch announced results in Physical Review Letters [**1**] for the scaling limit of the two point correlation function of the two dimensional Ising model. In [**19**] and [**20**] they published a full account of the remarkable result that this scaled correlation could be expressed in terms of a Painlevé function of the third kind. In a series of five long papers titled "Holonomic Quantum Fields I–V" published in the years 1978–1980, the mathematicians M. Sato, T. Miwa, and M. Jimbo (SMJ henceforth) revealed that the WMTB result was a specal case of a more general phenomena. There is a class of two dimensional quantum field theories whose correlation functions (i.e., Schwinger functions) could be expressed in terms of the solutions to nonlinear equations associated with monodromy preserving deformations of linear differential equations. SMJ named these quantum fields "Holonomic Quantum Fields" in reference to the intimate connection they have with holonomic systems of linear differential equations. A holonomic system of linear differential equations is one that is "maximally overdetermined" in a