PACIFIC JOURNAL OF MATHEMATICS
Vol. 160, No. 2, 1993

POSITIVE 2-SPHERES IN 4-MANIFOLDS OF SIGNATURE $(1, n)$

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We sharpen Donaldson's theorem on the standardness of definite intersection forms of smooth 4-manifolds in the same sense as Kervaire and Milnor sharpened Rohlin's signature theorem. We then apply the result thus obtained to show that the homology classes of rational surfaces with $b_2^- \leq 9$ which can be represented by smoothly embedded 2-spheres S with $S \cdot S > 0$ are up to diffeomorphism represented by smooth rational curves. Furthermore, we not only extend part of the application to the case where $b_2^- > 9$, but also give an algorithm to see whether or not a given homology class of rational surfaces with $b_2^- \leq 9$ can be represented by a smoothly embedded 2-sphere.

1. Introduction. Let M be a closed oriented smooth 4-manifold. One of the most important facts in 4-dimensional differential topology is the following:

THEOREM R (*Rohlin's signature theorem* [**13**]). *If the second Stiefel-Whitney class $w_2(M)$ vanishes, then the signature $\sigma(M)$ is congruent to* 0 *modulo* 16.

Performing the topological blowing up/down operations and applying Theorem R, Kervaire and Milnor [**6**] extended Theorem R to deduce the following:

THEOREM KM. *If an integral homology class ξ of M, dual to $w_2(M)$, is represented by a smoothly embedded 2-sphere in M, then the self-intersection number $\xi \cdot \xi$ must be congruent to $\sigma(M)$ modulo* 16.

Note that, although used in their proof of Theorem KM, Theorem R can be regarded as a special case of Theorem KM with $\xi = 0$.

The primary purpose of this paper is to sharpen the following in the same sense as Kervaire and Milnor sharpened Theorem R:

THEOREM D (*Donaldson* [**2**]). *If the intersection form of M is negative-definite ($b_2^+ = 0$), then it is equivalent over the integers to $\bigoplus b_2^-(-1)$.*