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A NOTE ON MORTON'S CONJECTURE CONCERNING THE LOWEST DEGREE OF A 2-VARIABLE KNOT POLYNOMIAL

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This note is concerned with the behaviour of the 'HOMFLY' polynomial of oriented links, $P_L(v, z)$. In particular, we show that the gap between the two lowest powers of v can be made arbitrarily large. This casts doubt on whether Morton's conjecture on the least v-degree can be established in general by the kind of combinatorial approach that has been successfully applied to some special cases.

Introduction. The two-variable knot polynomial $P_L(v, z)$ of a link L, announced in **[FYHLMO]**, **[PT]**, can be written in the form

$$P_L(v, z) = \sum_{i=e}^{E} a_i(z)v^i$$

where $a_i(z)$ is a polynomial in z for each i, $a_e(z) \neq 0$, and $a_E(z) \neq 0$. Let $f(P_L)$ denote the least degree in v in the polynomial P_L. Say that $f(P_L)$ is the *first* degree of P_L. With the above formulation $f(P_L) = e$. Let $s(P_L)$ be the least $i > e$ such that $a_i(z) \neq 0$. Say that $s(P_L)$ is the *second* degree of P_L.

In **[Mo3]** H. Morton conjectured that

$$f(P_L) \leq 1 - \chi(L)$$

for all links L where $\chi(L)$ is the maximum Euler characteristic over all orientable surfaces spanning L. In **[Cr]** I showed that the conjecture is satisfied by the homogeneous links (a class containing the positive and alternating links as special cases). A computer search for counterexamples in other classes of links showed up an interesting phenomenon: sometimes polynomials were produced where $s(P_L) - f(P_L)$ was quite large and $a_e(z) = 1$. In these cases it was only the term v^e, isolated from the other non-zero terms in the polynomial, which saved the conjecture from being violated. This prompted the question of whether $s(P_L) - f(P_L)$ could be arbitrarily large. Here I provide examples to show that it can.