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ON THE NON-OCCURRENCE OF THE COXETER GRAPHS β_{2n+1}, D_{2n+1} AND E_7 AS THE PRINCIPAL GRAPH OF AN INCLUSION OF II_1 FACTORS

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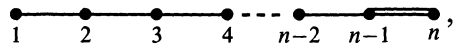
After discussing some preliminaries on the notion of an action of a hypergroup on a set, we present elementary proofs of the fact that the Coxeter graphs β_{2n+1}, D_{2n+1} and E_7 do not arise as Jones' principal graph invariant of an inclusion of II_1 factors. (Here, we use the symbol β_n to denote the graph that is normally denoted by B_n, the reason for this changed terminology being spelt out in the text.)

In this paper, we define and discuss some elementary consequences of the notion of an action of a hypergroup on a set and go on to use this notion to provide an elementary proof of the fact that the Coxeter graphs β_{2n+1}, D_{2n+1} and E_7 do not arise as Jones' principal graph invariant of an inclusion of II_1 factors. (The symbol β_n, rather than the symbol B_n, is used here to denote the graph



for the reason, pointed out to us by the referee, that the double bond acquires different meanings depending upon whether the graph is viewed as a Coxeter-Dynkin diagram or as a Bratteli diagram describing the inclusion of a pair of finite-dimensional C^*-algebras.)

The assertion about the D and E graphs was announced, but without proof, in [**O1**]. After the preparation of the manuscript, it was brought to the attention of the authors that the recent preprint [**I**] also contains a proof of the above facts about the D and E graphs, and that the preprint [**Ka**] proves the occurrence of the D_{2n} diagrams as well as uses Ocneanu's concept of a flat connection to demonstrate the non-occurrence of the D_{2n+1} graphs.

One reason for presenting our proof is that it is elementary, it shows the use of hypergroups as convenient book-keeping devices, and it can be read easily by one who is not too familiar with index-theory of subfactors of type III factors or the work of Longo in this direction