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BESOV SPACES, MEAN OSCILLATION, AND GENERALIZED HANKEL OPERATORS

MARCO M. PELOSO

We introduce some operators on the Bergman space A^2 on the unit ball that generalize the classical (big) Hankel operator. For such operators we prove boundedness, compactness, and Schatten-ideal property criteria. These extend known results. These new operators are defined in terms of a symbol. We prove in particular that for $2 \leq p < \infty$, these operators belong to the Schatten ideal S_p if and only if the symbol f is in the Besov space B_p. We also give several different characterizations of the norm on the Besov spaces B_p. In particular we prove that the Besov spaces are the mean oscillation spaces in the Bergman metric, for $1 \leq p < \infty$.

Consider the unit ball B in $\mathbf{C}^n$ and the Bergman space $A^2(B)$ of the holomorphic functions that are square integrable with respect to the (normalized) Lebesgue measure dm. The space A^2 admits a reproducing kernel $\mathscr{K}(z, w)$, the well-known Bergman kernel. Let $\mathscr{P}$ denote the orthogonal projection of $L^2(dm)$ onto A^2. The Hankel operator on A^2 with symbol f, H_f, is defined by

$$\begin{aligned} H_f g(z) &= (I - \mathscr{P})(\overline{f} g)(z) \\ &= \int_B \overline{(f(z) - f(w))} \mathscr{K}(z, w) g(w)\, dm(w). \end{aligned}$$

In recent times, the Hankel operator H_f first appeared in the context of Hardy spaces on the unit circle. It has been extensively studied by now, even on the Bergman spaces on the disc and on the ball. Important papers in this context are [**1**], [**3**] for the case $n = 1$, and [**2**], [**7**], [**19**], and [**21**] and [**23**] for the case $n > 1$. It is known that H_f is bounded if and only if f is in the Bloch space (see §2) and it is compact if and only if f is in the little Bloch space. These results are due to Axler for the case of the unit disc, and to Arazy, Fisher, Janson, and Peetre, and independently to Zhu, for the case of the ball. The Schatten ideal properties of H_f have been studied too. Arazy, Fisher, and Peetre for $n = 1$, [**3**], and independently four (groups of) authors ([**2**], [**7**], [**19**] and [**21**]) have proved that for $c(n) < p < \infty$, the Hankel operator H_f is in the Schatten von Neumann class S_p