PACIFIC JOURNAL OF MATHEMATICS
Vol. 161, No. 1, 1993

CONTACT STRUCTURES ON $(n-1)$-CONNECTED $(2n+1)$-MANIFOLDS

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A contact structure on a $(2n+1)$-dimensional manifold M is a completely non-integrable hyperplane distribution in the tangent bundle TM, i.e. a distribution which is (at least locally) defined by a 1-form α satisfying $\alpha \wedge (d\alpha)^n \neq 0$. An almost contact structure is a reduction of the structure group of TM to $\mathrm{U}(n) \times 1$. Every contact structure induces an almost contact structure.

Applying results of Eliashberg and Weinstein on contact surgery, we show that an $(n-1)$-connected $(2n+1)$-manifold is contact (to be precise: almost diffeomorphic to a contact manifold) if and only if it is almost contact.

1. Introduction. This paper is a sequel to [**4**], where we proved the following result.

THEOREM 1. *Let M be a simply-connected 5-manifold. Then M admits a contact structure in every homotopy class of almost contact structures.*

As in [**4**], all manifolds are assumed to be closed, oriented and smooth.

After the publication of [**4**], Eliashberg pointed out to me that he had obtained results similar to mine (but far more general) in [**1**]. We shall use his results to extend Theorem 1 to all $(n-1)$-connected $(2n+1)$-manifolds, which were classified by Wall [**10**] and Wilkens [**12**]. This classification is only up to almost diffeomorphism, that is, up to the connected sum with a homotopy sphere $\Sigma^{2n+1} \in \Theta_{2n+1}$, so the statement corresponding to Theorem 1 has to be weakened slightly in higher dimensions. Denote by bP_{2n+2} the subgroup of the group of homotopy spheres Θ_{2n+1} consisting of elements which bound a parallelizable manifold. Our extension of Theorem 1 can then be stated as

THEOREM 2. *Let M be an $(n-1)$-connected $(2n+1)$-manifold.*
If n is even (or $n = 1$), then M is almost diffeomorphic to a manifold M' which admits a contact structure in every homotopy class of almost contact structures.