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DEHN FUNCTIONS OF GROUPS AND EXTENSIONS OF COMPLEXES

STEPHEN G. BRICK

We study extensions of two-complexes and the Dehn functions (i.e. the isoperimetric inequalities) of their fundamental groups.

If $A \subset B$ are two complexes and their quotient X is diagramatically reducible then we obtain an upper bound for the Dehn function of $\pi_1(B)$ in terms of the Dehn functions of $\pi_1(A)$ and $\pi_1(X)$. In particular, we show that if the Dehn functions of $\pi_1(A)$ and $\pi_1(X)$ are bounded above by polynomials of degree n and m, then the Dehn function of $\pi_1(B)$ is bounded above by a polynomial of degree $n \cdot m$.

0. Introduction. In this paper, we look at extensions of two-complexes and the Dehn functions of their fundamental groups. We start by recalling some definitions from **[Br1]** (in addition, see **[Gr]**, **[Ge2]**, and **[CEHLPT]**). Let K be a finite two-complex. We will consider edge-paths (i.e. cellular maps of intervals into K) and will use $|*|$ for the length function. Write N_K for the set of edge-circuits that are null-homotopic in K. Given an integer $l \geq 0$, we set

$$N_K(l) = \{w \in N_K \mid |w| \leq l\}.$$

If $w \in N_K(l)$ then there is a Van-Kampen diagram for w, i.e. a pair $\mathbb{D} = (D, j)$ where $j: D \to K$ is a cellular map and D is a simply connected finite planar complex with boundary cycle $\partial\mathbb{D}$ mapping to w. Note that the boundary cycle $\partial\mathbb{D}$ is different from the topological boundary. In particular, as some edges appear twice, the length of the boundary cycle may be larger than the number of edges in the boundary.

Let $a(\mathbb{D})$ (also written $a(D)$) be the *combinatorial area* of $\mathbb{D}$, i.e. the number of faces of D. Set

$$\Delta_K(w) = \min\{a(\mathbb{D}) \mid \mathbb{D} \text{ is a diagram for } w\}.$$

We will say that a diagram $\mathbb{D}$ for w is *minimal* if $\Delta_K(w) = a(\mathbb{D})$. The *Dehn function* of K is the map $\delta_K: \mathbb{N} \to \mathbb{N}$, where $\mathbb{N} = \{0, 1, 2, \dots\}$, defined by

$$\delta_K(l) = \max\{\Delta_K(w) \mid w \in N_K(l)\}.$$