

## KNOTTING TRIVIAL KNOTS AND RESULTING KNOT TYPES

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Let  $(V, K)$  be a pattern (i.e.  $V$  is a standardly embedded solid torus in oriented  $S^3$  and  $K$  is a knot in  $V$ ) and  $f$  an orientation preserving embedding from  $V$  into  $S^3$  such that  $f(V)$  is knotted.

In this paper answers to the following questions will be given depending upon whether the winding number of  $K_2$  in  $V$  is zero or not.

(1) Suppose that  $K_1$  is unknotted and  $K_2$  is knotted in  $S^3$ . Can  $f(K_1)$  be ambient isotopic to  $f(K_2)$  in  $S^3$  for some embedding  $f: V \hookrightarrow S^3$ ?

(2) Suppose that  $K_1$  and  $K_2$  are both unknotted in  $S^3$ . How are  $(V, K_1)$  and  $(V, K_2)$  related if  $f(K_1)$  is ambient isotopic to  $f(K_2)$  in  $S^3$  for some embedding  $f: V \hookrightarrow S^3$ ?

**1. Introduction.** Let  $K$  be a knot in  $S^3$ , which is contained in a standardly embedded solid torus  $V$  ( $\subset S^3$ ). Assume that  $K$  is not contained in a 3-ball in  $V$ . Let  $f$  be an orientation preserving embedding from  $V$  into  $S^3$  such that  $f(C)$  is knotted in  $S^3$ , here  $C$  denotes a core of  $V$ . Then we get a new knot  $f(K)$  in  $S^3$  called a satellite knot with a companion knot  $f(C)$ . The knot  $K$  is called a preimage knot and we call the pair  $(V, K)$  a pattern (see Figure 1 on the next page).

Throughout this paper for an embedding  $f$  from  $V$  into  $S^3$ , we assume that it is orientation preserving and  $f(C)$  is knotted in  $S^3$ .

We concern ourselves with the following questions.

(1) Suppose that  $K_1$  is unknotted and  $K_2$  is knotted in  $S^3$ . Can  $f(K_1)$  be ambient isotopic to  $f(K_2)$  in  $S^3$  for some embedding  $f: V \hookrightarrow S^3$ ?

(2) Suppose that  $K_1$  and  $K_2$  are both unknotted in  $S^3$ . How are  $(V, K_1)$  and  $(V, K_2)$  related if  $f(K_1)$  is ambient isotopic to  $f(K_2)$  in  $S^3$  for some embedding  $f: V \hookrightarrow S^3$ ?

For two knots  $K_1$  and  $K_2$ , we write  $K_1 \cong K_2$  provided that there exists an orientation preserving self-homeomorphism of  $S^3$  carrying  $K_1$  to  $K_2$  (or equivalently,  $K_1$  and  $K_2$  are ambient isotopic in  $S^3$ ). For two patterns  $(V, K_1)$  and  $(V, K_2)$ , if there exists an orientation preserving self-homeomorphism  $h$  of  $V$  sending *longitude* to