PACIFIC JOURNAL OF MATHEMATICS
Vol. 161, No. 2, 1993

# ELLIPTIC REPRESENTATIONS FOR Sp(2n) AND SO(n)

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**Let $G$ be a connected, reductive $p$-adic group and let $G^e$ denote the set of regular elliptic elements of $G$. Let $\pi$ be an irreducible, tempered representation of $G$ with character $\Theta_\pi$, and write $\Theta_\pi^e$ for the restriction of $\Theta_\pi$ to $G^e$. We say $\pi$ is elliptic if $\Theta_\pi^e$ is non-zero. In this paper we will characterize the elliptic representations for the $p$-adic groups $\mathrm{Sp}(2n)$ and $\mathrm{SO}(n)$. We will show for $\mathrm{Sp}(2n)$ and $\mathrm{SO}(2n+1)$ that every irreducible, tempered representation is either elliptic or can be irreducibly induced from an elliptic representation. We will then show that this fails for the groups $\mathrm{SO}(2n)$. In this case there are irreducible tempered representations which cannot be irreducibly induced and are not elliptic.**

**Introduction.** For real reductive Lie groups, the elliptic representations are the discrete series and limits of discrete series representations. Knapp and Zuckerman [**K-Z**] classified the irreducible tempered representations by proving that every irreducible, tempered representation is either elliptic, or can be irreducibly induced from an elliptic representation of a proper parabolic subgroup in an essentially unique way. Thus the $p$-adic groups $\mathrm{Sp}(2n)$ and $\mathrm{SO}(2n+1)$ behave in the same way as real groups. In the $p$-adic case, Kazhdan [**K**] proved that an irreducible tempered representation is elliptic just in the case that it is not a linear combination (in the Grothendieck group) of properly induced representations. Clozel [**C**] conjectured that an irreducible tempered representation is elliptic, if and only if, it cannot be realized as a full induced representation from a proper parabolic subgroup. The case of $\mathrm{SO}(2n)$ provides a counterexample to Clozel's conjecture.

Every irreducible tempered representation is a subrepresentation of a representation unitarily induced from a discrete series representation of a parabolic subgroup. Thus in order to classify elliptic representations it is necessary to know which irreducible constituents of reducible induced representations are elliptic. In [**A**], Arthur gives such a characterization in terms of the $R$-group corresponding to the induced representation. In this paper we will use Arthur's results to characterize the elliptic representations of the symplectic and special