PACIFIC JOURNAL OF MATHEMATICS
Vol. 161, No. 2, 1993

JORDAN ANALOGS OF THE BURNSIDE AND JACOBSON DENSITY THEOREMS

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If $\mathscr{A}$ is an (associative) algebra of linear operators on a vector space, it is well known that 2-transitivity for $\mathscr{A}$ implies density and, in certain situations, transitivity guarantees 2-transitivity. In this paper we consider analogs of these results for Jordan algebras of linear operators with the standard Jordan product.

0. Introduction. Let $\mathscr{L}(\mathscr{V})$ be the algebra of all linear operators on a vector space $\mathscr{V}$ over the field $\mathbb{F}$. A subset $\mathscr{S}$ of $\mathscr{L}(\mathscr{V})$ is called transitive if $\mathscr{S}x = \mathscr{V}$ for every nonzero x in $\mathscr{V}$. More generally, $\mathscr{S}$ is called k-transitive if given linearly independent vectors $x_1, x_2, \ldots, x_k$ and arbitrary vectors $y_1, y_2, \ldots, y_k$ in $\mathscr{V}$ there exists a member S of $\mathscr{S}$ such that $Sx_i = y_i$, $i = 1, 2, \ldots, k$. If $\mathscr{S}$ is k-transitive for every k, then it is called (strictly) dense. It is a remarkable fact due to Jacobson [2] that if $\mathscr{S}$ is an (associative) subalgebra of $\mathscr{L}(\mathscr{V})$, then 2-transitivity implies density for arbitrary $\mathbb{F}$. In particular, if $\mathscr{V}$ is finite-dimensional, then $\mathscr{L}(\mathscr{V})$ is the only 2-transitive algebra on $\mathscr{V}$. There are transitive algebras that are not 2-transitive even if $\mathbb{F}$ is algebraically closed. In the presence of certain conditions (e.g., topological) transitivity implies density. The most well-known result of this kind is Burnside's theorem [3]: if $\mathscr{V}$ is finite-dimensional and $\mathbb{F}$ is algebraically closed, then the only transitive algebra over $\mathscr{V}$ is $\mathscr{L}(\mathscr{V})$.

In this paper we consider analogs of these results for Jordan algebras of operators: linear spaces $\mathscr{A}$ of operators such that A^2 and ABA belong to $\mathscr{A}$ for all A and B in $\mathscr{A}$. If the characteristic of the field $\mathbb{F}$ is different from 2, this is equivalent to the requirement that $\mathscr{A}$ be closed under the Jordan bracket $\{A, B\} = AB + BA$. Over this kind of field a Jordan algebra $\mathscr{A}$ may be equivalently defined as a linear space closed under taking positive integral powers. For the sake of completeness we include proofs of a few elementary facts obtainable from the general theory of Jordan algebras [4].

In what follows we often find it convenient to view members of $\mathscr{L}(\mathscr{V})$ as matrices over $\mathbb{F}$; this should cause no confusion. The set