PACIFIC JOURNAL OF MATHEMATICS
Vol. 161, No. 2, 1993

THE TEMPERLEY-LIEB ALGEBRA AT ROOTS OF UNITY

FREDERICK M. GOODMAN AND HANS WENZL

We present general techniques to determine the structure of Hecke algebras and similar algebras in the non-semisimple case. We apply these to give a complete description of the structure of the Temperley-Lieb algebras at a root of unity. Our description implies in particular that the representation of these algebras on tensor space $(\mathbf{C}^2)^{\otimes n}$ is faithful.

Introduction. We shall consider ascending sequences of finite dimensional algebras $A_1 \subset A_2 \subset \cdots$, given by generators and relations, where the relations depend on one or several parameters. Moreover,we also assume that the discriminants of these algebras are non-zero polynomials or rational functions in the parameters. This means the algebras are semisimple except for special values of the parameters. For applications (to the construction of topological invariants, the construction of subfactors, or in statistical mechanical models) these algebras are often needed at the critical values of the parameters; in such cases, interesting semisimple quotients have been constructed in [J1], [W2,4].

In this paper, we initiate a systematic study of the structure of such algebras at the critical values of the parameters, where they are not semisimple.

For the examples we have in mind such as Hecke algebras, Brauer algebras, etc., the structure is known in the semisimple case and can be described by the Bratteli diagram, which encodes how an irreducible representation of A_n, restricted to A_{n-1}, decomposes into irreducible representations of A_{n-1}. If all the multiplicities in the decompositions are 0 or 1, one can use this to define special path idempotents and matrix units (see e.g. [SV], [W1,2], [RW]), labelled by paths on the Bratteli diagram. These matrix units are only well defined for generic values of the parameters, for which the algebras are semisimple. Nevertheless, their (usually inductive) defining formulas carry a lot of information about the structure of the algebras which can also be exploited at the critical parameter values. In more detail, our main techniques are as follows: