BIHOLOMORPHIC CONVEX MAPPINGS OF BALL IN $\mathbb{C}^n$

Sheng Gong, Shikun Wang, and Qihuang Yu

Biholomorphic convex mappings from unit ball in $\mathbb{C}^n$ into $\mathbb{C}^n$ are studied in this paper. A Schwartz type lemma for the class of mappings and a necessary and sufficient condition under which a holomorphic mapping is biholomorphic and convex are established. The results are used to describe some characteristics of the image of the class of mappings.

1. Introduction. Since Loebe discovered as early as 1907 his "Verzerrugsatz", classical distortion theorems for families of univalent functions defined the unit disc in the complex plane $\mathbb{C}$ have developed systematically in depth and scope. For several variables, H. Cartan showed his interest in the field and conjectured [1] that the magnitude of the determinant of the complex Jacobian of a normalized biholomorphic mapping on the unit polydisc in $\mathbb{C}^2$ should have a finite upper and a positive lower bound. But, it was pointed out in [2] that the conjecture is not correct. That is the distortion theorem for general biholomorphic mappings in several variables does not hold. This suggests one has to find some of their subclasses for which the distortion theorem can still hold. In 1988 S. Gong, C. H. FitzGerald and R. W. Bernard [3] obtained first time the upper and lower bound of the magnitude of the determinant of the Jacobian of a normalized, convex and biholomorphic mapping from the unit ball $B = \{Z = (z^1, z^2) \in \mathbb{C}^2 | |Z| < 1\}$ into $\mathbb{C}^2$. After that Taishun Liu [4] generalized the result to the unit ball in $\mathbb{C}^n$ and X. A. Zheng [5] to the bounded symmetric domains.

It is worth pointing out that the estimate of the magnitude of the determinant of the Jacobian of a holomorphic mapping is one of the generalization of distortion theorems of one variable. The estimation of eigenvalues of the Hermitian matrix, the product of the complex Jacobian of a holomorphic mapping and its transpose conjugate, should also be considered to be another form of generalization of distortion theorems of one complex variable. In this contribution we will discuss the type of distortion theorems and its application to biholomorphic convex mappings.