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ON THE METHOD OF CONSTRUCTING IRREDUCIBLE FINITE INDEX SUBFACTORS OF POPA

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Let $U^s(Q)$ be the universal Jones algebra associated to a finite von Neumann algebra Q and $R^s \subset R$ be the Jones subfactors, $s \in \{4\cos^2\frac{\pi}{n} | n \geq 3\} \cup [4, \infty)$. We consider for any von Neumann subalgebra $Q_0 \subset Q$ the algebra $U^s(Q, Q_0)$ defined as the quotient of $U^s(Q)$ through its ideal generated by $[Q_0, R]$ and we construct a Markov trace on $U^s(Q, Q_0)$. If $\mathscr{Z}(Q) \cap \mathscr{Z}(Q_0) = \mathbb{C}$ and Q contains $n \geq s+1$ unitaries $u_1 = 1, u_2, \ldots, u_n$, with $E_{Q_0}(u_i^* u_j) = \delta_{ij}1$, $1 \leq i, j \leq n$, then we get a family of irreducible inclusions of type II_1 factors $N^s \subset M^s$, with $[M^s : N^s] = s$ and minimal higher relative commutant. Although these subfactors are nonhyperfinite, they have the Haagerup approximation property whether $Q_0 \subset Q$ is a Haagerup inclusion and if either Q_0 is finite dimensional or $Q_0 \subset \mathscr{Z}(Q)$.

Introduction. Let M be a finite factor with the normal finite faithful trace τ and denote by $L^2(M, \tau)$ the completion of M in the Hilbert norm $\|x\|_2 = \tau(x^*x)^{1/2}$, $x \in M$. For $N \subset M$ subfactor of M $(1_N = 1_M)$, the Jones index $[M; N]$ is defined as the Murray-von Neumann coupling constant $\dim_N L^2(M)$ of N in its representation on the Hilbert space $L^2(M, \tau)$. Jones **[J]** proved that $[M : N]$ can only take the values $\{4\cos^2\frac{\pi}{n} | n \geq 3\} \cup [4, \infty]$ and constructed a one parameter family R^s of subfactors of the hyperfinite type II_1 factor R with $[R : R^s] = s$, $s \in \{4\cos^2\frac{\pi}{n} | n \geq 4\} \cup [4, \infty)$.

When $s = [M : N] = 4\cos^2\frac{\pi}{n}$, $n \geq 3$, the properties of the local index **[J]** imply that the pair $N \subset M$ is irreducible (i.e. $N' \cap M = \mathbb{C}$). For $s \geq 4$ Jones' inclusions $R^s \subset R$ are reducible and the problem of characterizing the values $s \geq 4$ with the property that there exist inclusions $R_0 \subset R$ with $[R : R_0] = s$ and $R_0' \cap R = \mathbb{C}$ remained open.

The problem of finding all possible values of indices of irreducible finite index subfactors in arbitrary II_1 factors was completely answered by Popa, who constructed in **[P2]** irreducible inclusions of nonhyperfinite type II_1 factors $N^s \subset M^s$, with $[M^s : N^s] = s$, for all $s \in \{4\cos^2\frac{\pi}{n} | n \geq 4\} \cup [4, \infty)$. His method consists in constructing certain traces, that he called Markov traces, on some universal algebras $U^s(Q)$ canonically associated with a given finite von Neumann algebra Q and