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VERTEX OPERATOR CONSTRUCTION OF STANDARD MODULES FOR $A_n^{(1)}$

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We generalize the vertex operator formula for the affine Lie algebra $A_n^{(1)}$ in the "homogeneous picture" and by using it we construct a basis of any given standard $A_n^{(1)}$-module parametrized by coloured partitions. We also obtain a similar explicit construction of vacuum spaces of standard $A_2^{(1)}$-modules.

1. Introduction. In this paper we give an explicit construction of standard (i.e. integrable highest weight) representations of affine Lie algebra $\tilde{\mathfrak{g}}$ of the type $A_n^{(1)}$.

As usual, for $\mathfrak{g} = \mathfrak{sl}(n+1, \mathbf{C})$ we fix a Cartan subalgebra $\mathfrak{h}$ and root vectors x_α, and we identify $\mathfrak{h} \cong \mathfrak{h}^*$ via bilinear form $\langle x, y\rangle = \operatorname{tr} xy$. We denote by c the canonical central element of the affine Lie algebra $\tilde{\mathfrak{g}}$ and we write $x(i) = x \otimes t^i$ for $x \in \mathfrak{g}$ and $i \in \mathbf{Z}$. As usual we use triangular decompositions

$$\mathfrak{g} = \mathfrak{n}_- + \mathfrak{h} + \mathfrak{n}_+, \qquad \tilde{\mathfrak{g}} = \tilde{\mathfrak{n}}_- + \tilde{\mathfrak{h}} + \tilde{\mathfrak{n}}_+.$$

Let $\mathfrak{n}_0 \subset \mathfrak{n}_+$ be the nilpotent radical of a maximal parabolic subalgebra of $\mathfrak{g}$ such that its Levi factor is (isomorphic to) $\mathfrak{gl}(n, \mathbf{C})$. Let $\Gamma = \{\gamma_1, \dots, \gamma_n\}$ be the set of weights of $\mathfrak{n}_0$ (see §2). Then

$$\{x_\beta(j);\ \beta \in \Gamma,\ j \in \mathbf{Z}\}$$

is a commutative family in $\tilde{\mathfrak{g}}$.

Let $L(\Lambda)$ be a standard $\tilde{\mathfrak{g}}$-module with a highest weight vector v_Λ. On $L(\Lambda)$ we have a projective representation $\beta \mapsto e_\beta$ of the root lattice Q of $\mathfrak{g}$ (see §5). Let

$$x_\alpha(\zeta) = \sum_{j \in \mathbf{Z}} x_\alpha(j)\zeta^j.$$

By using the formal Laurent series technique we extend the vertex operator formula for level 1 $A_n^{(1)}$-modules and for level $k \geq 1$ $A_1^{(1)}$-modules to all standard $A_n^{(1)}$-modules, based on a simple observation that the vertex operator formula for level 1 representation can